Summer School « Non-homogeneous fluids and flows », Prague, August 27-31, 2012

Nonlinear properties of internal gravity waves
Part II : non-homogeneous medium

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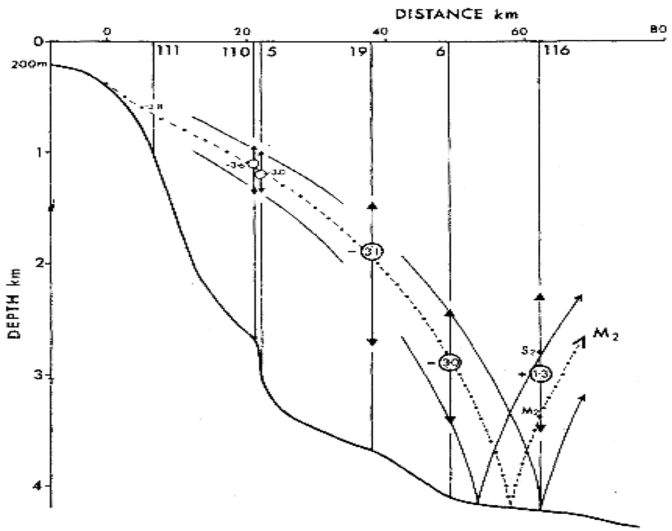
Introductory remarks

As opposed to what I showed in my first lectures, internal gravity waves rarely propagate in a quiescent medium with uniform N.



New and Pingree 1992

Actually, at the end of spring or in summer, a seasonal thermocline forms in the upper ocean upon which an internal wave beam may reflect. What does happen then ?
This question received attention recently, as solitary waves observed in the middle of the ocean, far from the coast, are supposed to result from such an interaction.

→ This problem will be addressed in the first part of the lectures.

Introductory remarks (cont'd)

Another common inhomogeneity of the medium occurs when the medium is moving at velocity U(**x**,t).

The simplest case occurs for a steady shear flow, such as a horizontal wind varying along the vertical, U(z). This problem was addressed for the first time by Bretherton in the 60's (using ray theory).
The case of a horizontal shear flow has been addressed a little later, by Morozov in 1974 (using ray theory again).
→ I shall review the main properties of a wave field propagating in a shear flow, as well as discuss about energy exchanges between the wave field and the mean flow.

Finally, nonlinear internal gravity waves generate a mean flow, which is irreversible and cumulative in time if the medium is dissipative.
The important rôle of such a mean flow has been realized in the atmosphere but nothing has been done along this direction in the ocean.
→ I shall illustrate such a mean flow generation at the end of the lectures.

Outline

I. Propagation of internal gravity waves in a non homogeneous fluid
(in a background N(z) stratification)
(coll : Nicolas Grisouard and Theo Gerkema)

II. Propagation of internal gravity waves in a non homogeneous flow
(in a background U(y,z) velocity field)
- Propagation in a horizontal mean flow with a vertical shear
- Propagation in a horizontal mean flow with a horizontal shear

III. Wave-induced mean flow
- Nonlinear reflexion of an internal gravity wave on a uniform slope
(coll : Louis Gostiaux, Nicolas Grisouard and Matthieu Leclair)
- Deep ocean dynamics in the Southern Ocean
(coll : Pierre Labreuche and Julien Le Sommer)

I. Propagation of internal gravity waves in a non homogeneous fluid (in a background N(z) stratification)

Interaction of a wave beam with a seasonal thermocline

coll : Nicolas Grisouard and Theo Gerkema

« Local generation » of internal solitary waves

New & Pingree 1990: in the Bay of Biscay (BoB), solitary waves arise (too) far from the continental slope

Explanation (New & Pingree 1990): IW beam impinging on the thermocline ⇒ "local" generation.

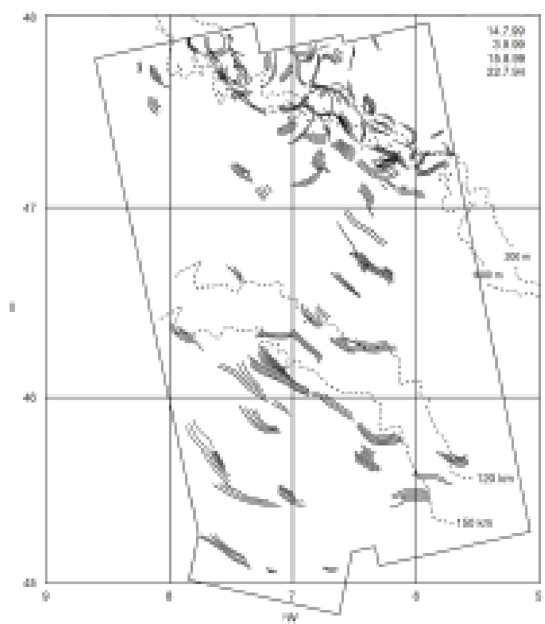


Figure: SAR images of the BoB gathered (New & Da Silva 2002)

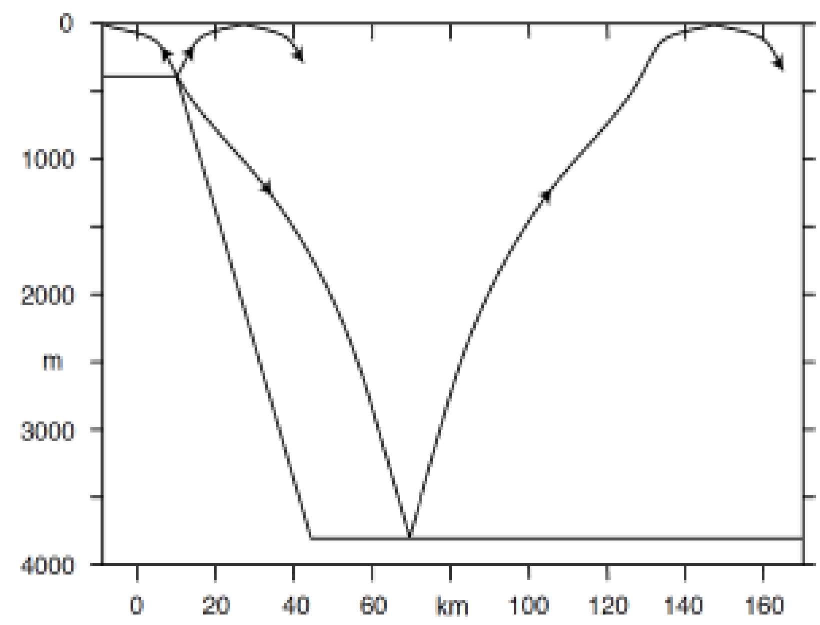


Figure: Ray paths in the BoB near 47°N (New & Da Silva 2002)

Previous works

- Observations in the ocean

  Konyaev 1995 (Indian Ocean), New & Pingree 1990, 1991, 1992 (BoB), New & Da Silva 2002 (BoB), Da Silva et al 2007 (Portugal), Da Silva et al. 2008 (Mozambique Channel)

- Theoretical models

  Delisi & Orlanski (1975), Thorpe (1998), Gerkema (2001), Akylas et al (2007)

But no (nonlinear non-hydrostatic) numerical simulation or laboratory experiment showing that such a generation process is possible.

→ This is the purpose of what follows.

Numerical set-up

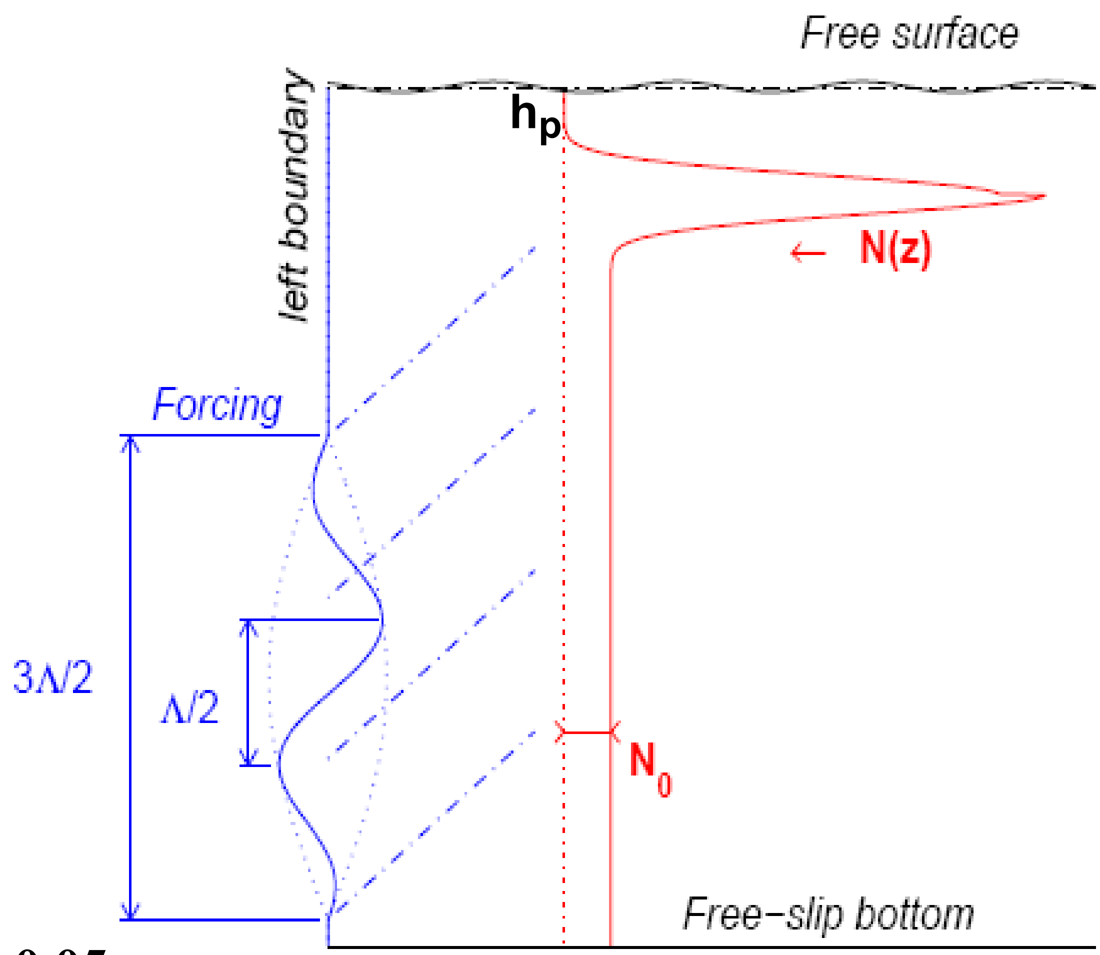
Code: MITgcm. **(Boussinesq, NL, NH)**

Stratification:

1. upper layer, $N = 0$,
2. thermocline,
3. lower layer, constant N_0.

Forcing of the velocity on the boundary, vertical scale Λ.

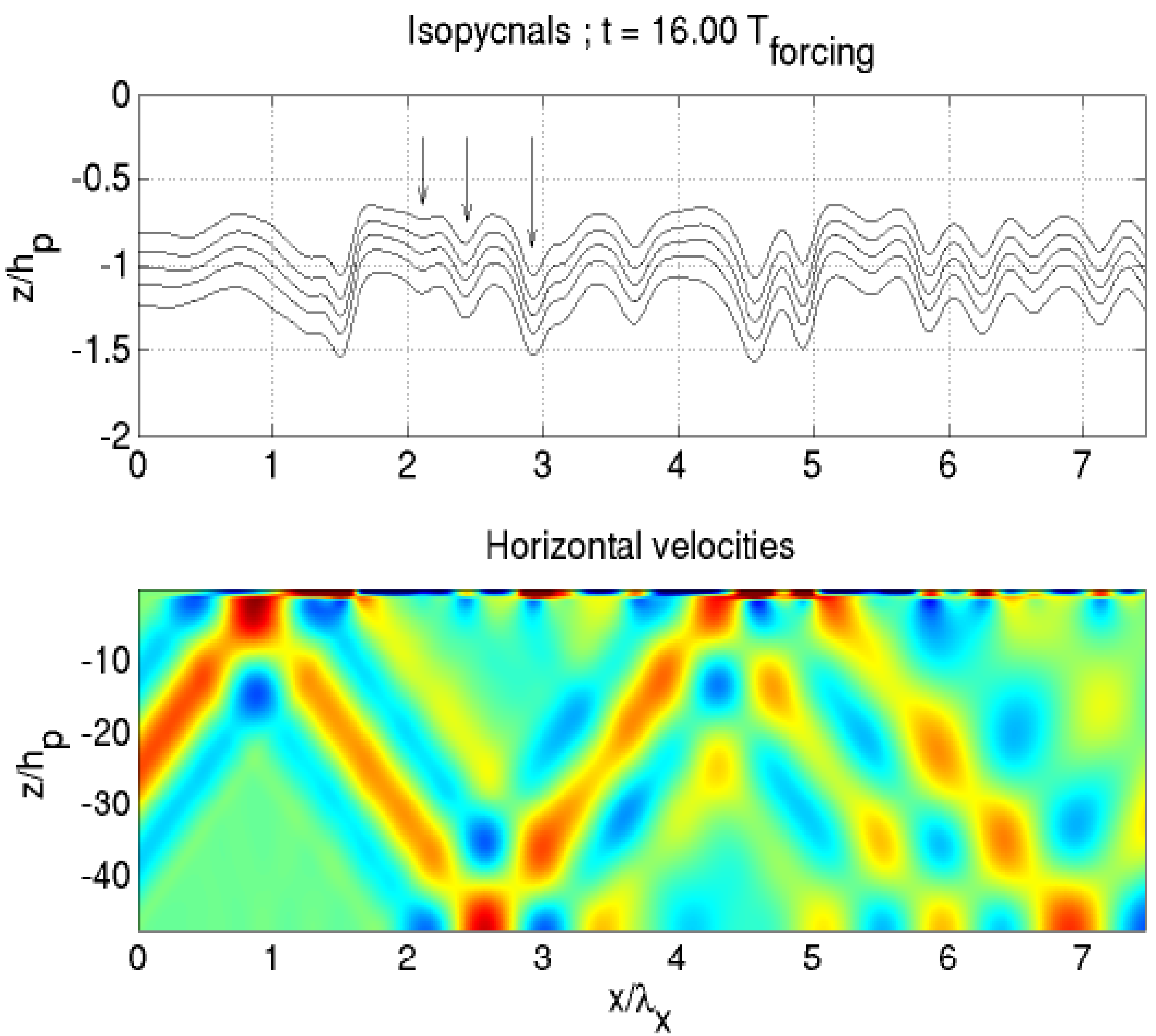
No rotation.



Grid lepticity (dx/h_p)=0.2→0.05

Generation of mode-1 internal solitary waves

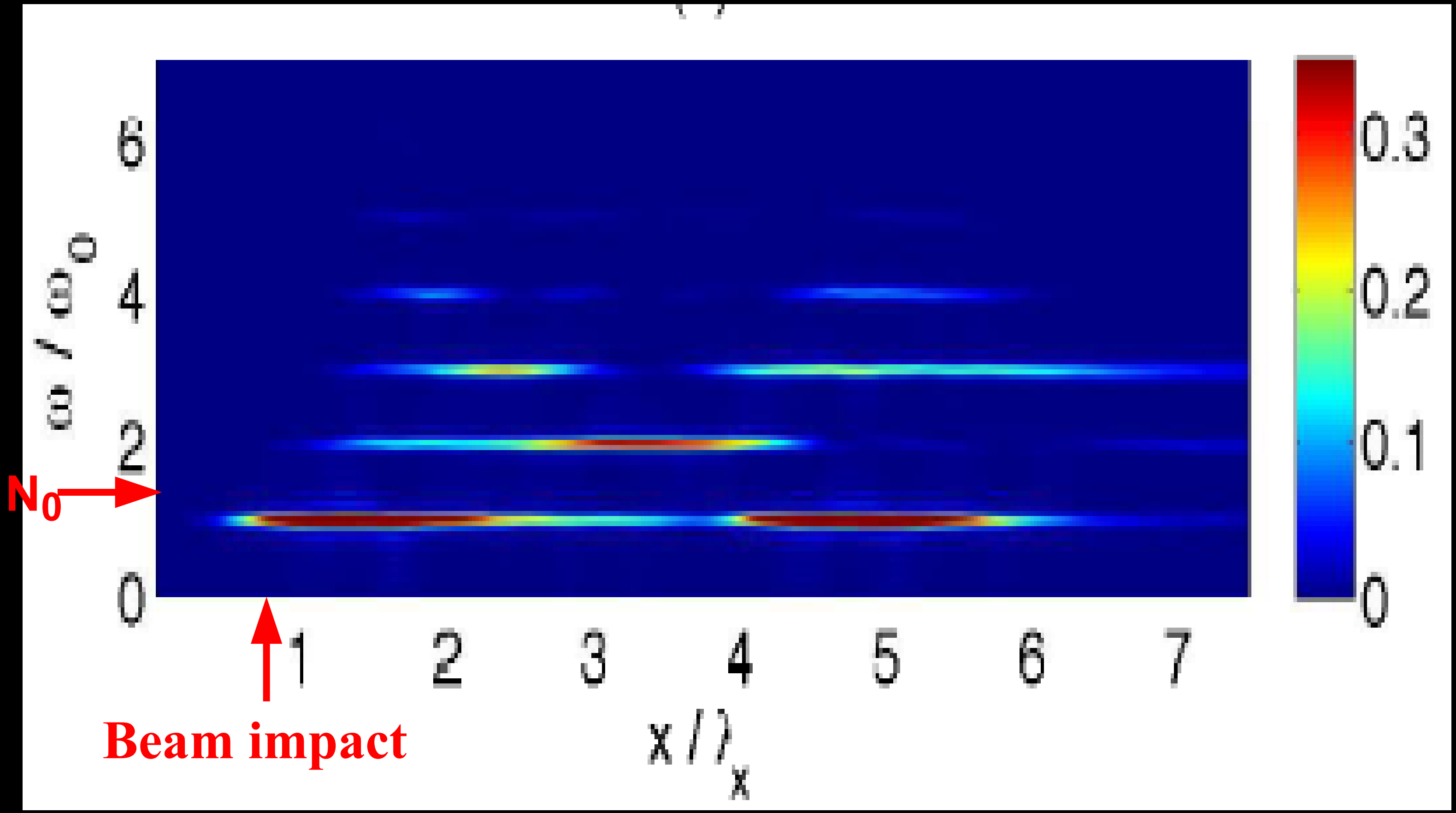
h_p: thickness of the homogeneus layer, λ_x: horizontal wavelength of the wave beam



Development of harmonics in the thermocline

Displacement of the isotherm at the center of the thermocline :
Space-frequency contour plot

(ω_0 : forcing frequency of the wave beam, λ_x : horizontal wavelength of the wave beam)



An empirical criterion for the generation of internal solitary waves

1. Assumption :
   The generation process of the internal solitary wave (by the impinging of the wave beam on the thermocline) is linear;
   → the internal solitary wave evolves from a linear internal gravity wave (« the thermocline wave »); how can this occur ?

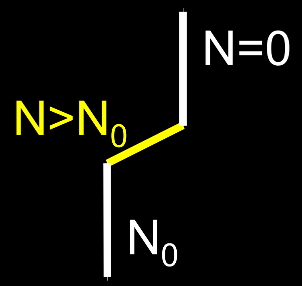
2. Simple remarks :
   The resonant forcing of a temporal oscillator occurs when
   frequency of the forcing = natural frequency of the oscillator
   The resonant forcing of a (1D) spatial oscillator occurs when
   wavelength of the forcing = natural wavelength of the oscillator

→ For a propagating oscillator (the thermocline wave), resonant forcing occurs if
phase speed of the forcing = natural phase speed of the excited wave.

3. This equality was shown by Delisi & Orlanski (1975) and Akylas et al. (2007) to maximize the displacement of a thermocline of zero thickness (an interface) hit by a plane wave or by a beam.

An empirical criterion for the generation of internal solitary waves (cont'd)

→ What about a thermocline of finite thickness ?



• Finite thickness → there can be different wave modes in the thermocline.
• We assume that for a thermocline wave of mode n to be excited, one must have

horizontal phase speed of the wave beam ≈ phase speed of mode n

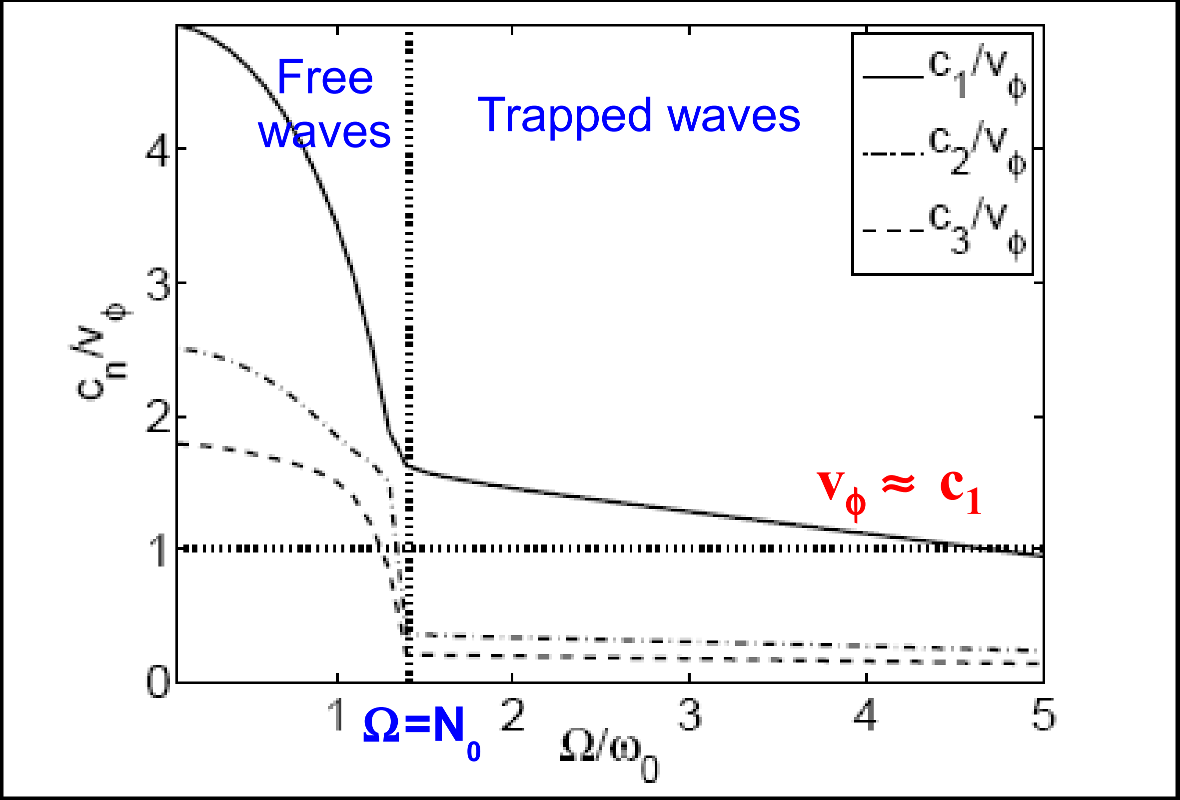
$$V_\phi \approx c_n$$

→ How to compute the phase speed of a mode-n thermocline wave ?
Search for a solution $w(x,z,t) = W(z) \exp[i(Kx-\Omega t)]$ in the linear B. equations, which yields the equation for the (W_n, c_n) (modal decomposition) :

$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{c_n^2} W_n = 0 \quad + B.Cs. \quad (n = 1, 2, \dots) \qquad (c_n = \Omega/K_n)$$

An empirical criterion for the generation of internal solitary waves (cont'd)

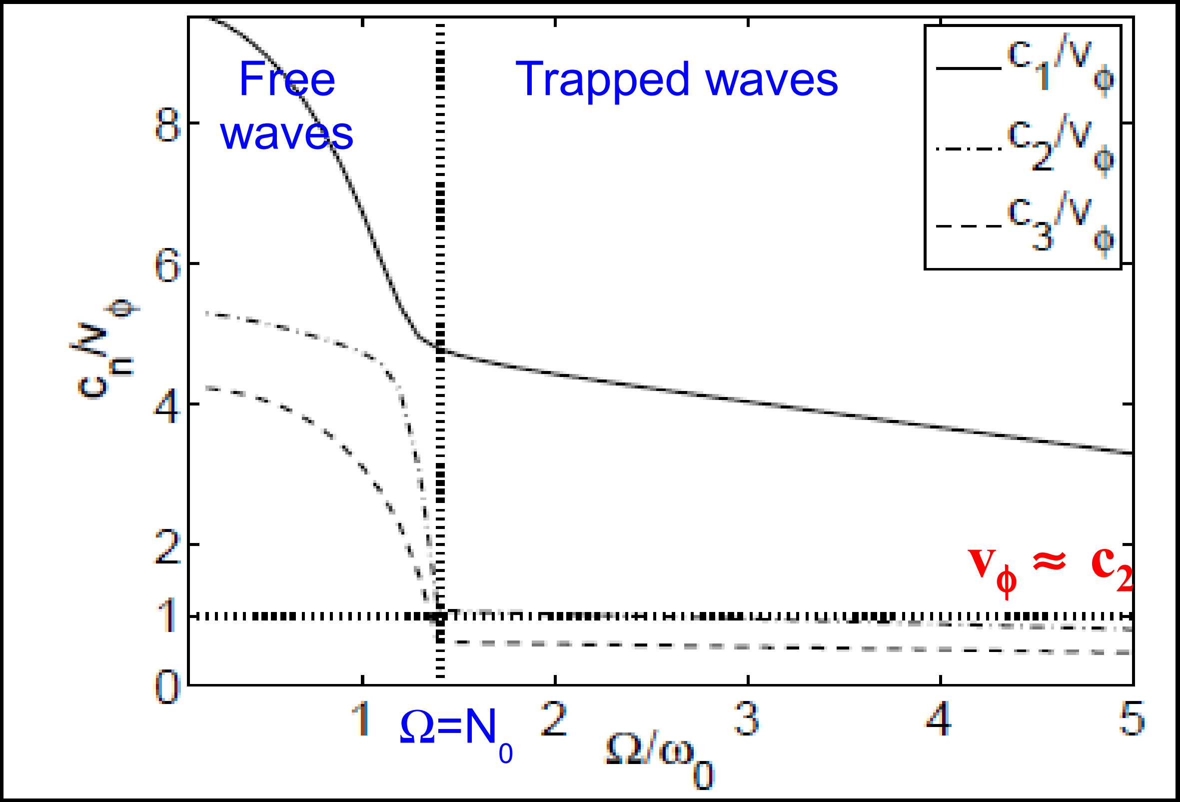
$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{\mathbf{c}_n^2} W_n = 0 \quad + B.Cs. \quad (n = 1, 2, \ldots)$$



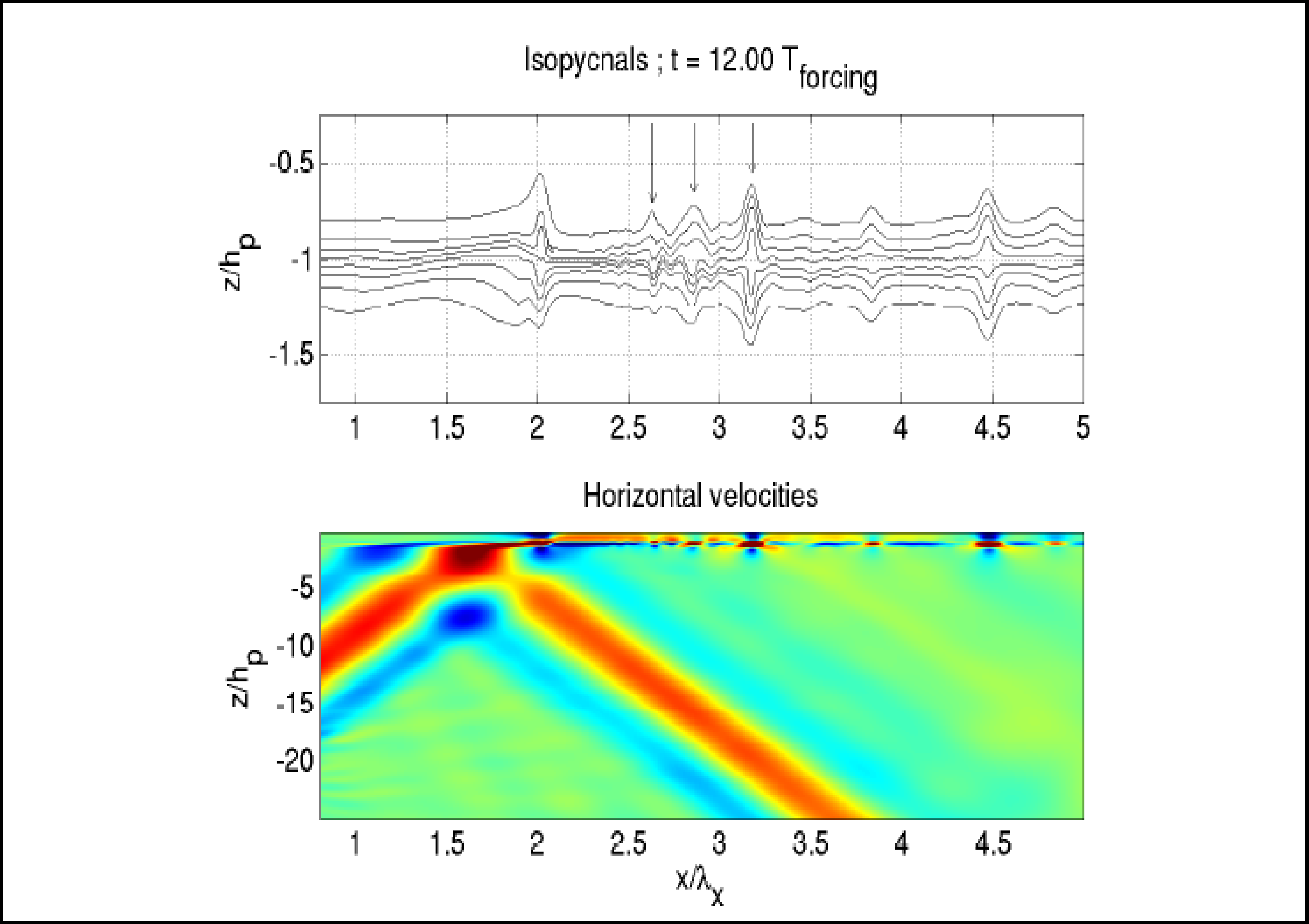
→ this is how the numerical simulation was designed, choosing N(z) and the beam structure such that $\mathbf{v}_\phi = \mathbf{c}_1$

An empirical criterion for the generation of internal solitary waves (cont'd)

$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{\mathbf{c}_n^2} W_n = 0 \quad + B.Cs. \quad (n = 1, 2, \dots)$$



Application to mode-2 internal solitary waves: numerical experiment



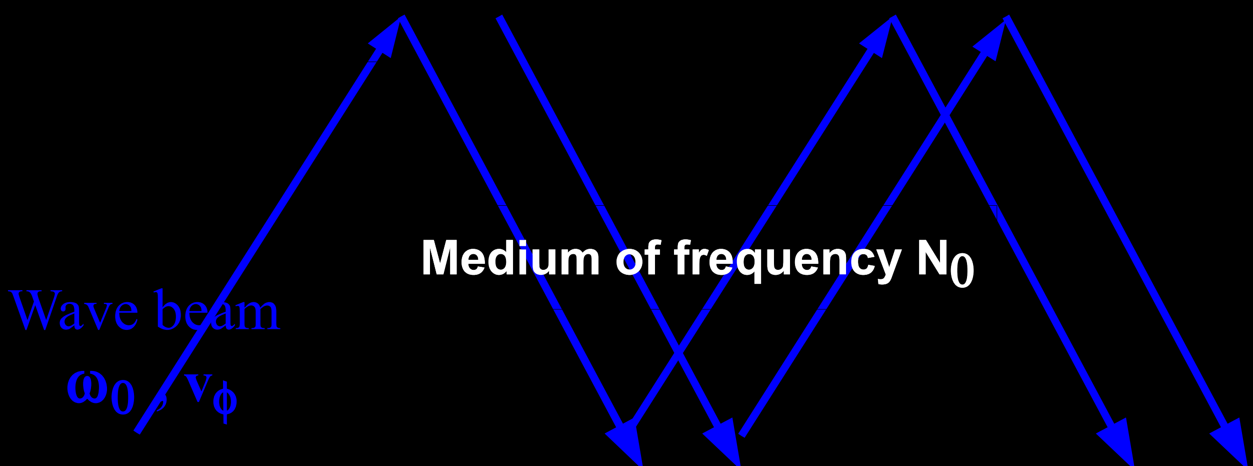
Summary

Thermo-
cline

Free linear wave
ω_0 , $c_n \neq v_\phi$

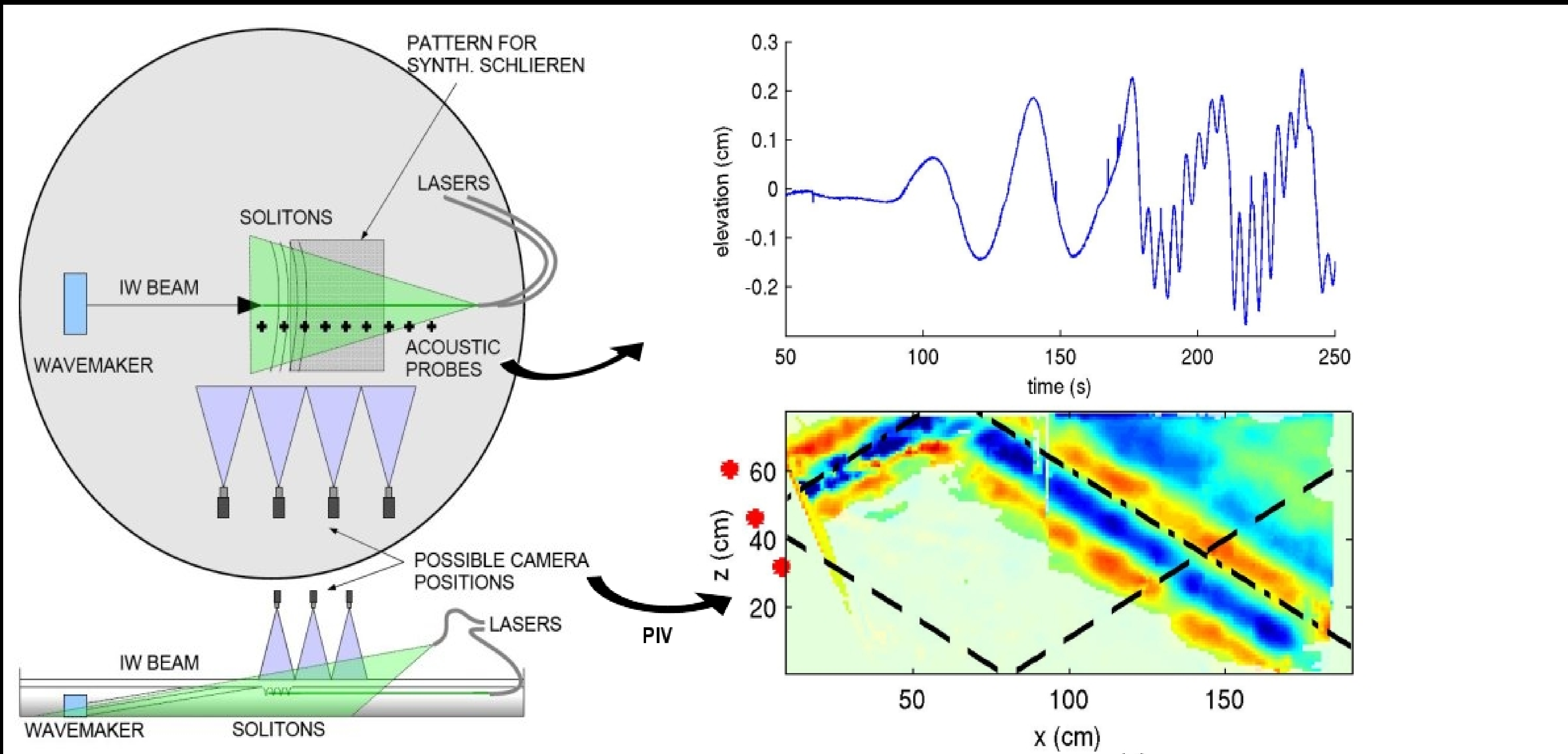
Quasi-linear trapped wave
$n\omega_0 > \mathbf{N_0}$, $c_n \approx v_\phi$

Nonlinear
(solitary) wave



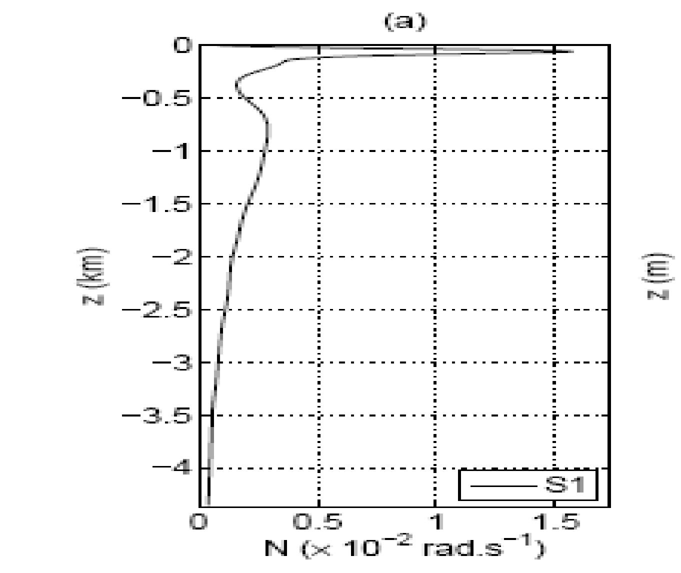
Grisouard, Staquet & Gerkema, 2011, J. Fluid Mech.
Grisouard & Staquet, 2010, Nonlin. Proc. Geophys.

« Local generation » of internal solitary waves : laboratory experiments

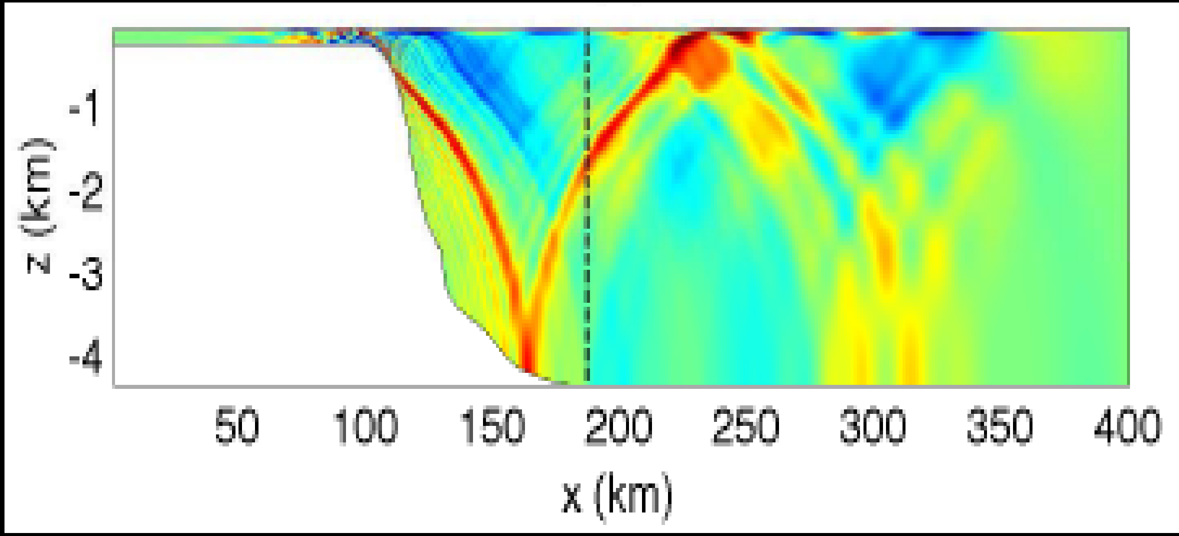


Mercier, Mathur, Gostiaux, Gerkema et al. 2012 J. Fluid Mech.

Toward the real world : local generation of internal solitary waves in the Bay of Biscay



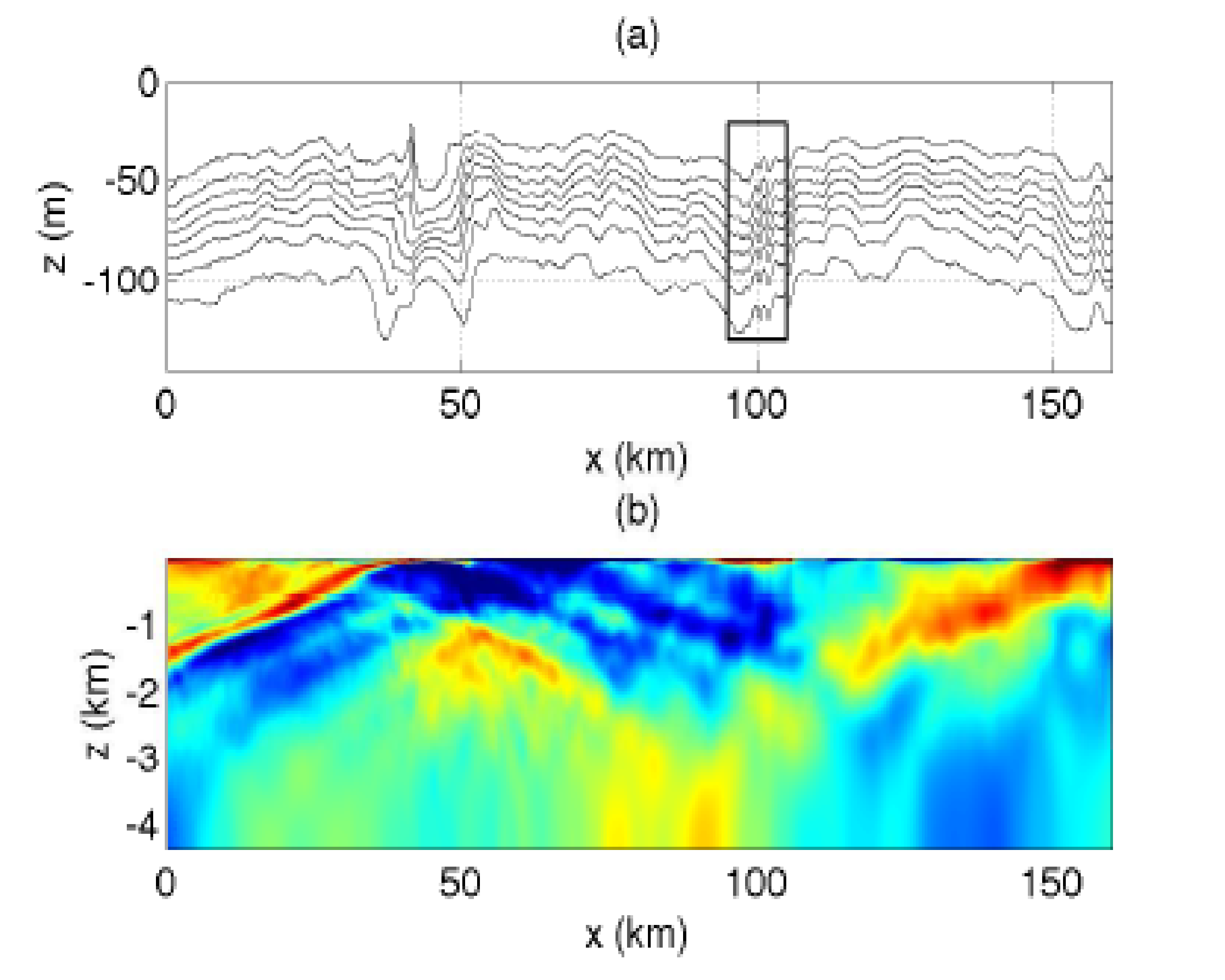
Initial Brunt-Vaisala profile



Forcing

Toward the real world : local generation of internal solitary waves in the Bay of Biscay

Generation of mode-1 internal solitary waves in the thermocline



Grisouard & Staquet, 2011, Nonlin. Proc. Geophysics

II. Interaction of an internal gravity wave with a mean flow

(non dissipative, linear and steady regime)

Interaction of an internal gravity wave with a mean flow

A brief introduction on ray theory

We consider a slowly varying medium [U(y,z), N(y,z)] with respect to wave characteristics along the y- and z-directions. That is, the medium is locally homogeneous and the wave may be assumed to be locally plane.

The ray theory predicts the wave geometry along a ray : $dx/dt = \mathbf{c_g} + \mathbf{U}$, where $\mathbf{c_g}$ is the group velocity, namely (for $\mathbf{U}=U\mathbf{e}_x$).

$dk_x/dt = 0$

$dk_y/dt = - \partial\Omega/\partial N \;\; \partial N/\partial y - k_x \, \partial U/\partial y$

$dk_z/dt = - \partial\Omega/\partial N \;\; \partial N/\partial y - k_x \, \partial U/\partial z$

$d\omega/dt = 0$, with $\omega = \Omega + \mathbf{k.U}$ where ω is the absolute frequency and Ω, the intrinsic frequency

$d/dt=\partial/\partial t + (\mathbf{c_g} + \mathbf{U}).\nabla$

In the following, 'ray theory' → 'WKB approximation'.

Interaction of an internal gravity wave with a mean flow

Equation for the wave action

(Garrett 1968)

- We consider a plane monochromatic internal gravity of frequency Ω propagating in an inviscid fluid at rest.

The wave-induced energy E satisfies the conservation equation :

$\partial E/\partial t+\nabla.(\mathbf{c}_g E)=0$, where $\mathbf{c}_g$ is the group velocity

- If the fluid is moving at velocity $\mathbf{U}(\mathbf{x},t)$, the wave-induced energy is no longer conserved.

Within the WKB approximation, the wave action $A=E/\Omega$ is conserved :

$$\partial A/\partial t+\nabla.[\, (\mathbf{c}_g+U)A \,] = 0$$

(with Ω and E being the *intrinsic* frequency and wave-induced energy, i.e. measured within the moving system).

→ **Since the wave-induced energy E is no longer conserved, energy can be transferred between the wave and the mean flow.**

Interaction of an internal gravity wave with a mean flow

Vertical shear flow U(z)

We assume that $Ri = N^2/(dU/dz)^2 > 1/4$, with N constant.

From WKB approximation, along a ray :

- Since the properties of the medium do not vary in time and along the x-direction : ω (the absolute frequency) and k_x are constant along a ray (i.e. $d\omega/dt=0$ and $dk_x/dt=0$).
- $dk_z/dt = - k_x\, dU/dz$ and $d\Omega/dz = - k_x\, dU/dz$ (Ω : intrinsic frequency)

$\rightarrow$ Two cases, depending on the sign of $k_x\, dU/dz$

Interaction of an internal gravity wave with a mean flow

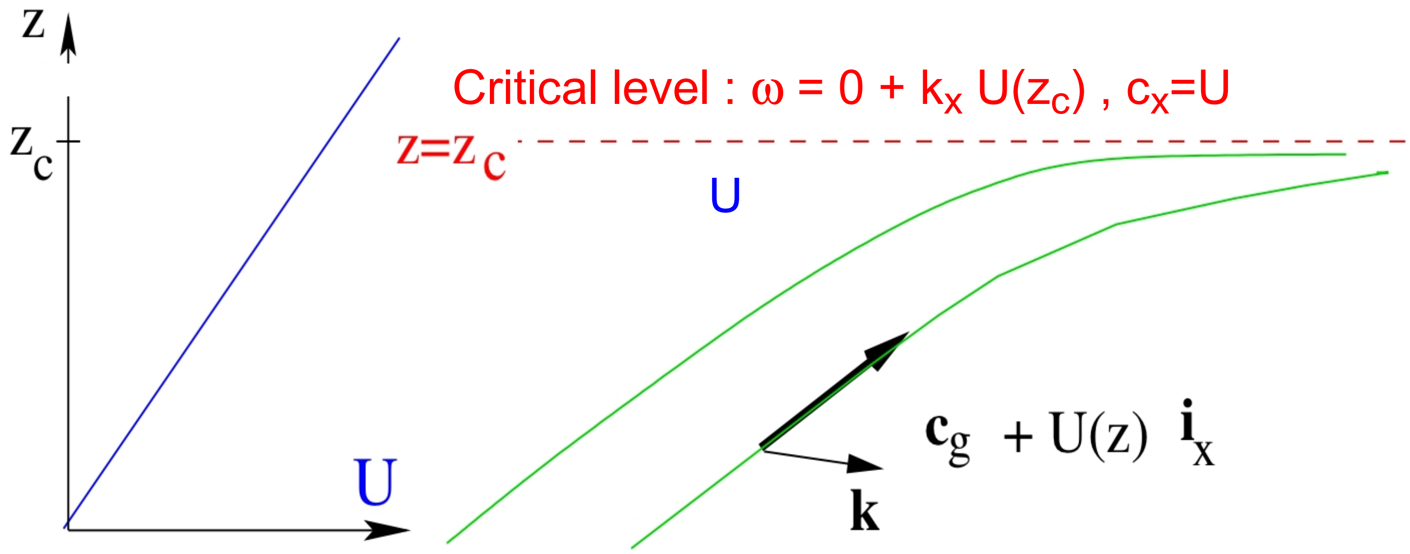
Vertical shear flow U(z)

Case 1 : k_x dU/dz >0

We assume that the wave propagates upwards in a shear flow $U(z)\mathbf{e}_x$ with dU/dz >0.
Then along a ray : $|k_z| \uparrow$; Ω and $|\mathbf{c_g}| \downarrow$.

If there exists a level z_c such that $\Omega=0$ («critical level»), the wave is trapped at that level.

The WKB approximation predicts that this level is reached in an infinite time.
What does actually happen ?



Interaction of an internal gravity wave with a mean flow

Vertical shear flow U(z)

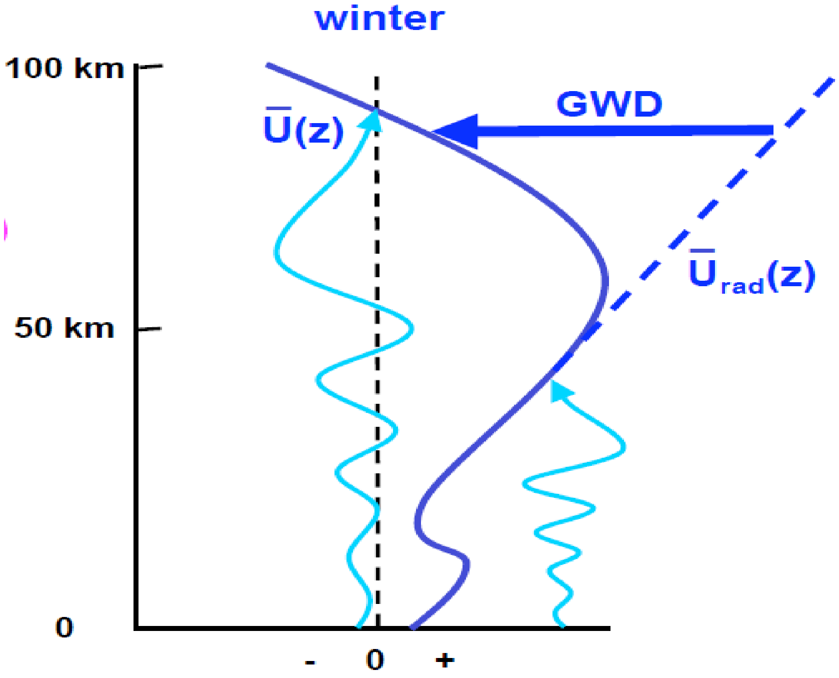
<u>Case 1 : k_x dU/dz >0 (continued)</u>

Relaxing WKB approximation, using the linearized inviscid Boussinesq equations for Ri>1/4, Booker & Bretherton (1967) show that, as the wave propagates through the critical level, the wave-induced momentum flux is attenuated by a factor exp[-2π (Ri-1/4)$^{1/2}$] :

Hence, if Ri>>1/4, there is no transmitted wave and the wave is absorbed by the mean flow.

If wave-induced momentum flux **F** is opposed to **U** (**F.U**<0), **the mean flow is decelerated at the critical level.** **<u>This is the case for lee waves.</u>**

Fundamental process in the atmosphere :



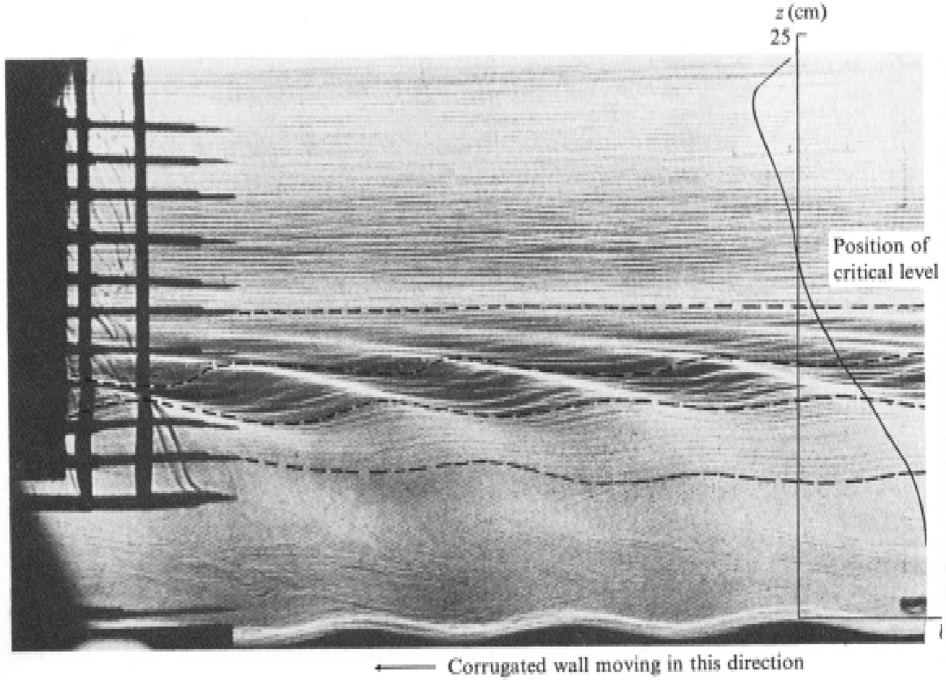
Fritts, 2010

Interaction of an internal gravity wave with a mean flow

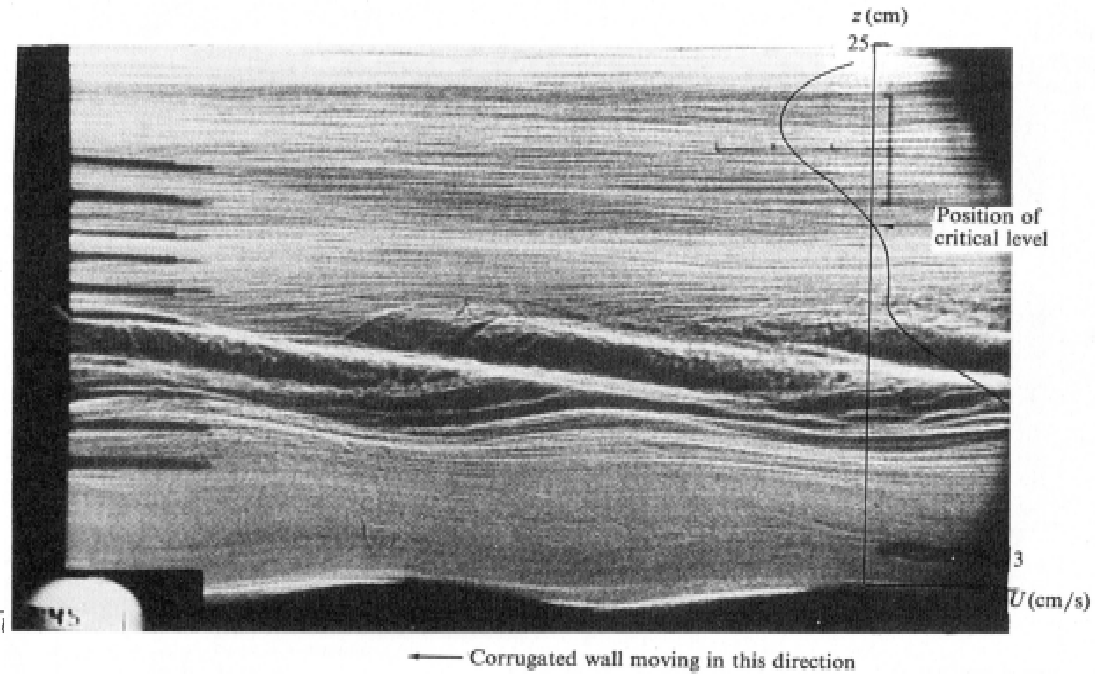
Vertical shear flow U(z)

Case 1 : k_x dU/dz >0 (continued)

Koop & McGee (Journal of Fluid mechanics, 1986)



Weak amplitude waves (and Ri=40): no transmission across the critical level



Strong amplitude waves (and Ri=4): breaking below the critical level

Breaking despite of absorption ? When $z \rightarrow z_c$, $u'_{max} \rightarrow 0$, $c_x \rightarrow 0$ but c_x decreases faster than u'_{max} so that $s = u'_{max}/c_x$, which is one measure of wave slope, increases (McIntyre 2000).

3D numerical experiments by Winters & D'Asaro (1994) for large amplitude waves and Ri=1/2 : 35% incident wave energy is reflected, 35% is absorbed (mean flow acceleration) and the remainder is dissipated into turbulence and mixes the fluid.

Interaction of an internal gravity wave with a mean flow

Vertical shear flow U(z)

Case 2 : k_x dU/dz < 0

Ri>1/4

Ω (intrinsic frequency) increases and may reach N : the wave reflects at that level.

Ri<1/4 (Jones 1968)

The shear flow is unstable and, if a reflecting level exists (Ω=N), overreflection occurs.

Interaction of an internal gravity wave with a mean flow

Horizontal shear flow U(y)

From WKB approximation, along a ray (N constant):

- Since the properties of the medium do not vary in time and along the x- and z-directions : ω (the absolute frequency) and k_x and k_z are constant along a ray (i.e. $d\omega/dt=0$, $dk_x/dt=0$ and $dk_z/dt=0$).

- $dk_y/dt = - k_x \, dU/dy$ and $d\Omega/dy = - k_x \, dU/dy$ (Ω : intrinsic frequency)

$\rightarrow$ Two cases, depending on the sign of $k_x \, dU/dy$.

Interaction of an internal gravity wave with a mean flow

Horizontal shear flow U(y)

Case 1 : $k_x\, dU/dy < 0$

We assume that the wave enters into the shear flow with dU/dy >0.
Then along a ray : Ω and $|k_y| \uparrow$; $|\mathbf{c_g}| \downarrow$.

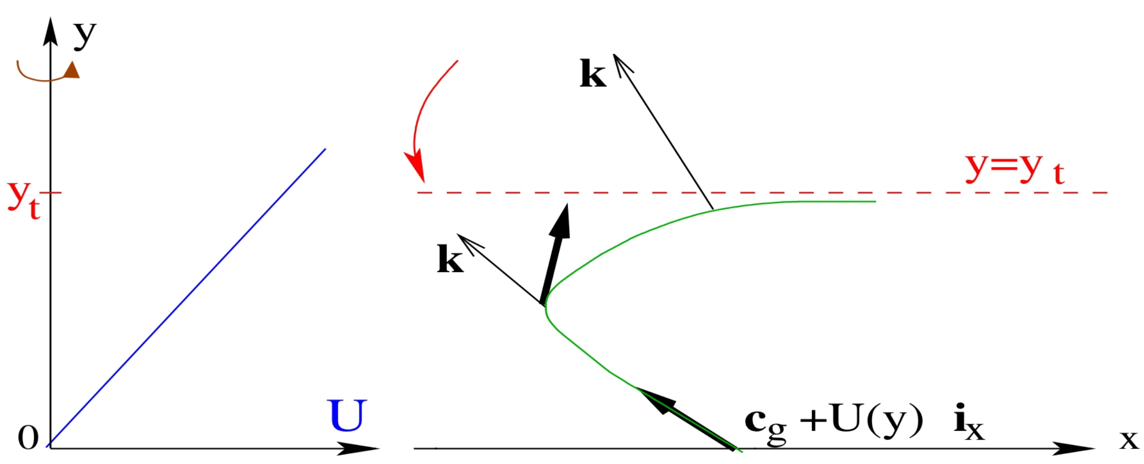
If there exists y_t such that $\Omega = N$ («trapping plane»), the wave is trapped at that level.

From WKB approximation : energy is transferred from the shear flow to the wave at the trapping level $\rightarrow$ wave amplification occurs.

Can the wave break at that level ?

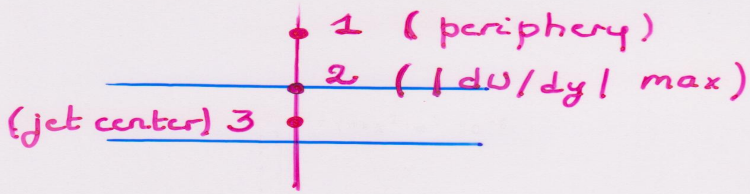
Trapping plane : $\omega = N + k_x\, U(y_t)$

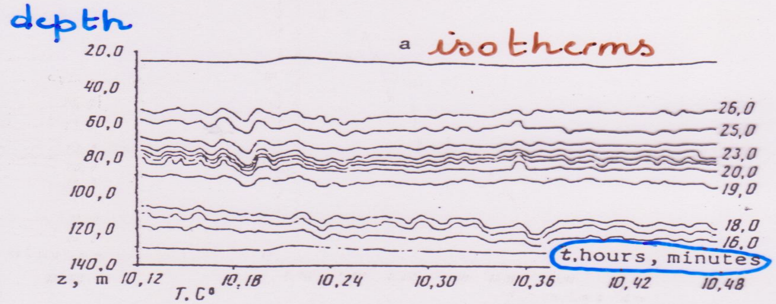
Horizontal (x,y) plane

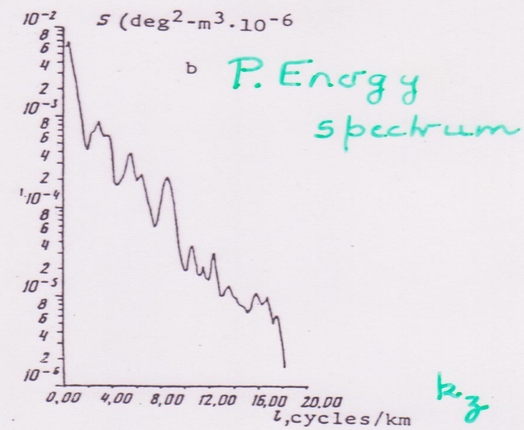


Ivanov & Morozov 1974, Badulin et Tsimring 1984, Olbers 1981, Badulin & Shrira 1993

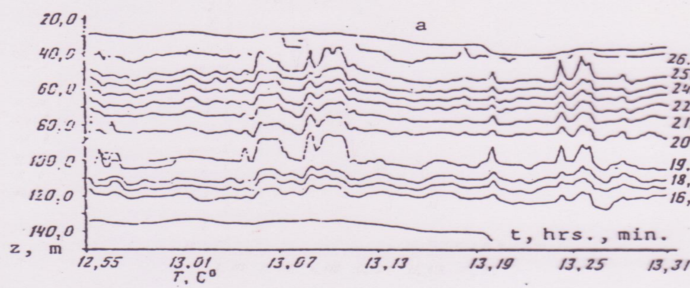
From "Transformation of internal waves in the equatorial Lomonosov current", Badulin et al. ('90)

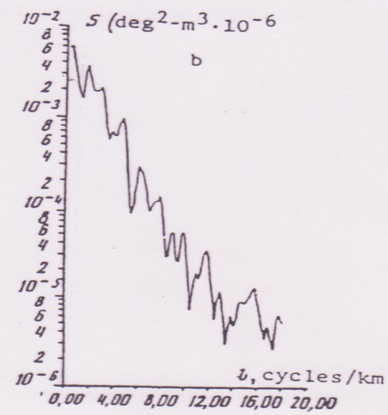




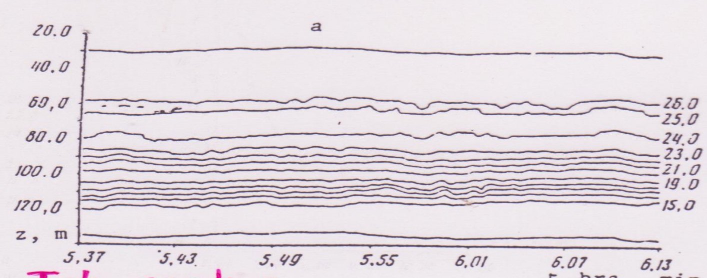


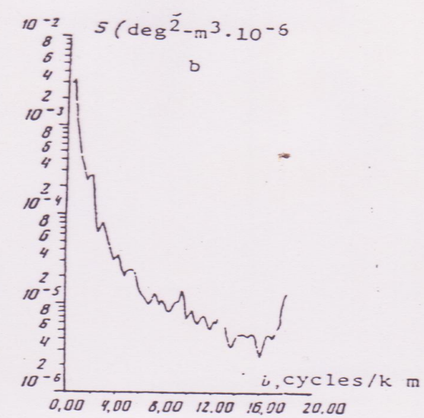
1. Periphery of the jet.





2. |dU/dy| max





3. Jet center

III. Wave-induced mean flow

Mean flow induced by a dissipative internal gravity wave field

Up to now, a wave (or wave packet) propagated in an ambient shear flow.

What does happen if the medium at rest ?

Since a monochromatic plane wave is a solution of the inviscid Boussinesq equations, it has no net effect on the medium.

→ Non-acceleration theorem (Andrews et al. 1987) : If the wave field is both steady, linear and non-dissipative, no mean flow changes are induced.

By contrast, a weakly nonlinear wave field induces a mean flow, which is irreversible if the medium is dissipative.

The induced mean flow is second order in the wave amplitude (namely $U \sim s^2$).

Mean flow induced by a dissipative internal gravity wave field

It is well-known that waves propagating in a dissipative medium induce a mean flow (through the attenuation of the wave-induced Reynolds stress) :
This is the sonic wind for sound waves f.i. (see Lighthill 1978, McIntyre 2000)

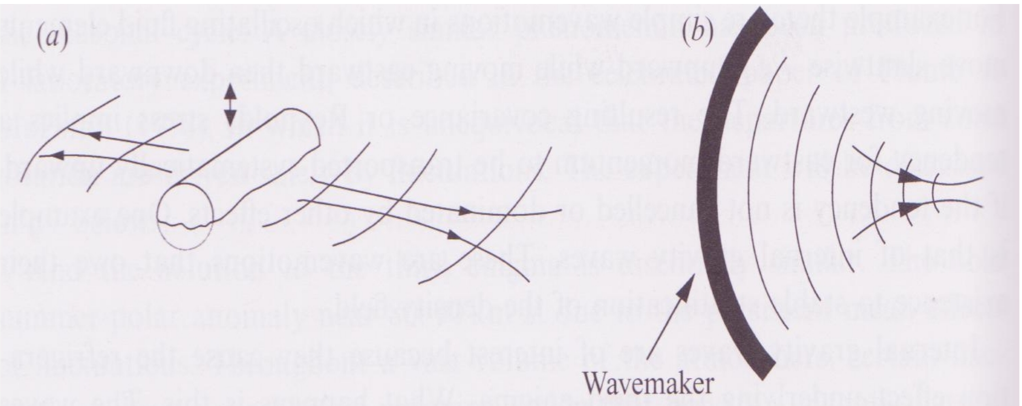


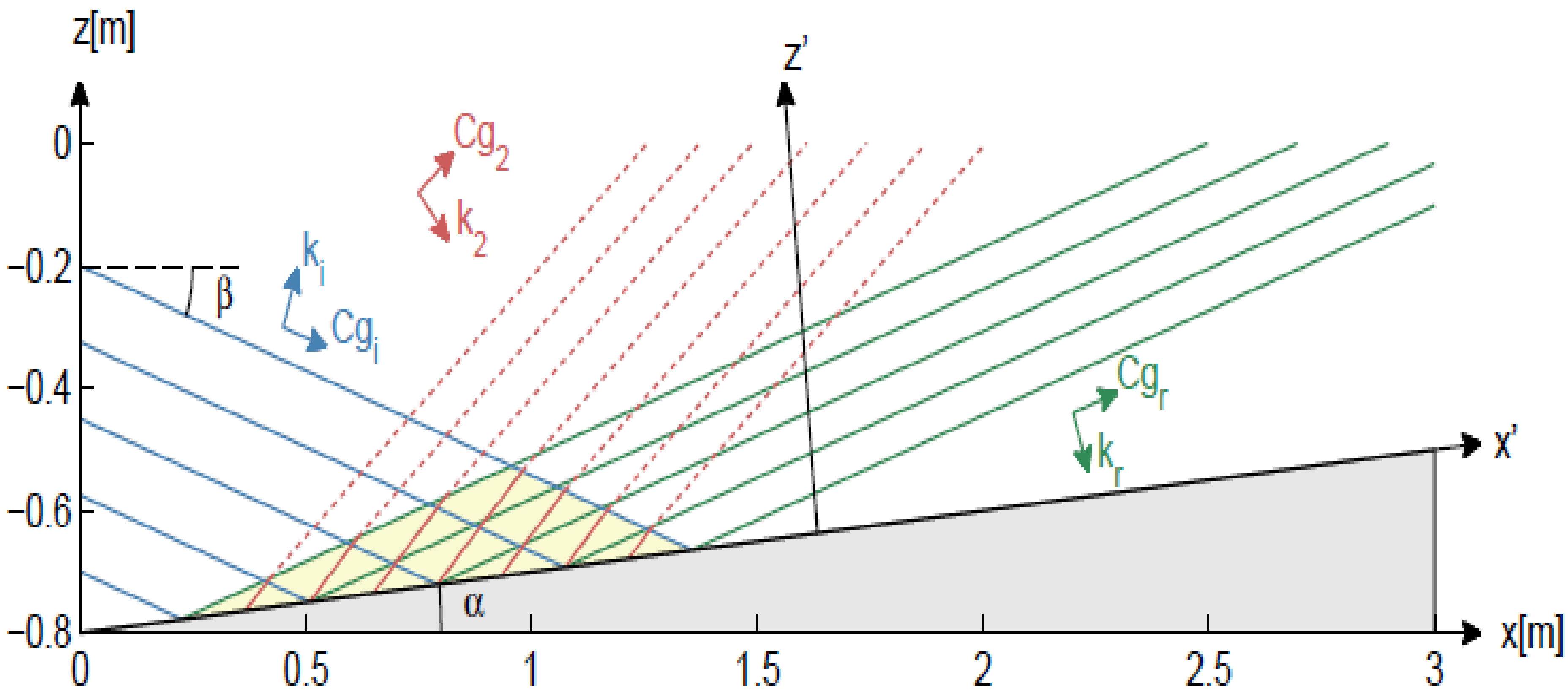
Figure 3. Simple experiments with surface waves on water, illustrating wave-induced momentum transport, generating a strong mean flow that can be made visible by sprinkling a little powder such as chalk dust on the surface of the water. Experiment (*a*), my standard lecture demonstration, uses a cylinder about 10 cm long and 4 cm in diameter. In experiment (*b*), the curved wavemaker is about 60 cm in arclength and radius. Good results are obtained with capillary–gravity waves at frequencies around 5Hz.

McIntyre 2000

The wave-induced mean flow may be seen as the result of a force exerted by the waves on the fluid.
For waves in a 2D vertical plane, the associated acceleration is : $\mathbf{dU/dt = -\,\partial(u'w')/\partial z}$.

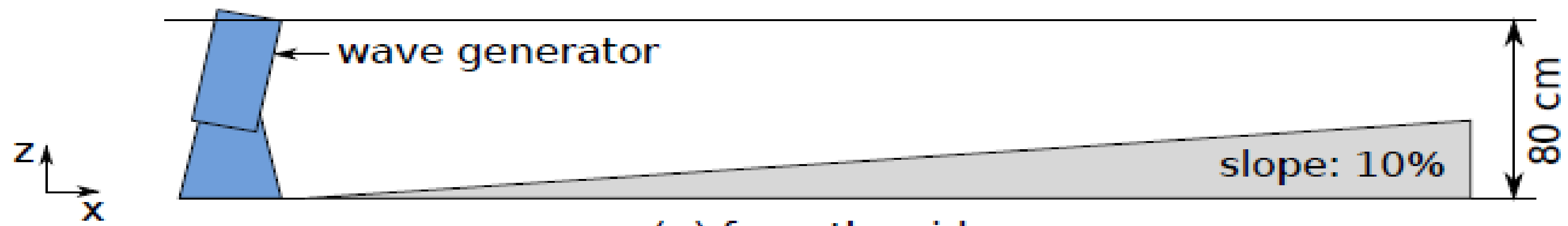
Nonlinear reflexion of internal gravity waves on a uniform slope

Sketch of the configuration

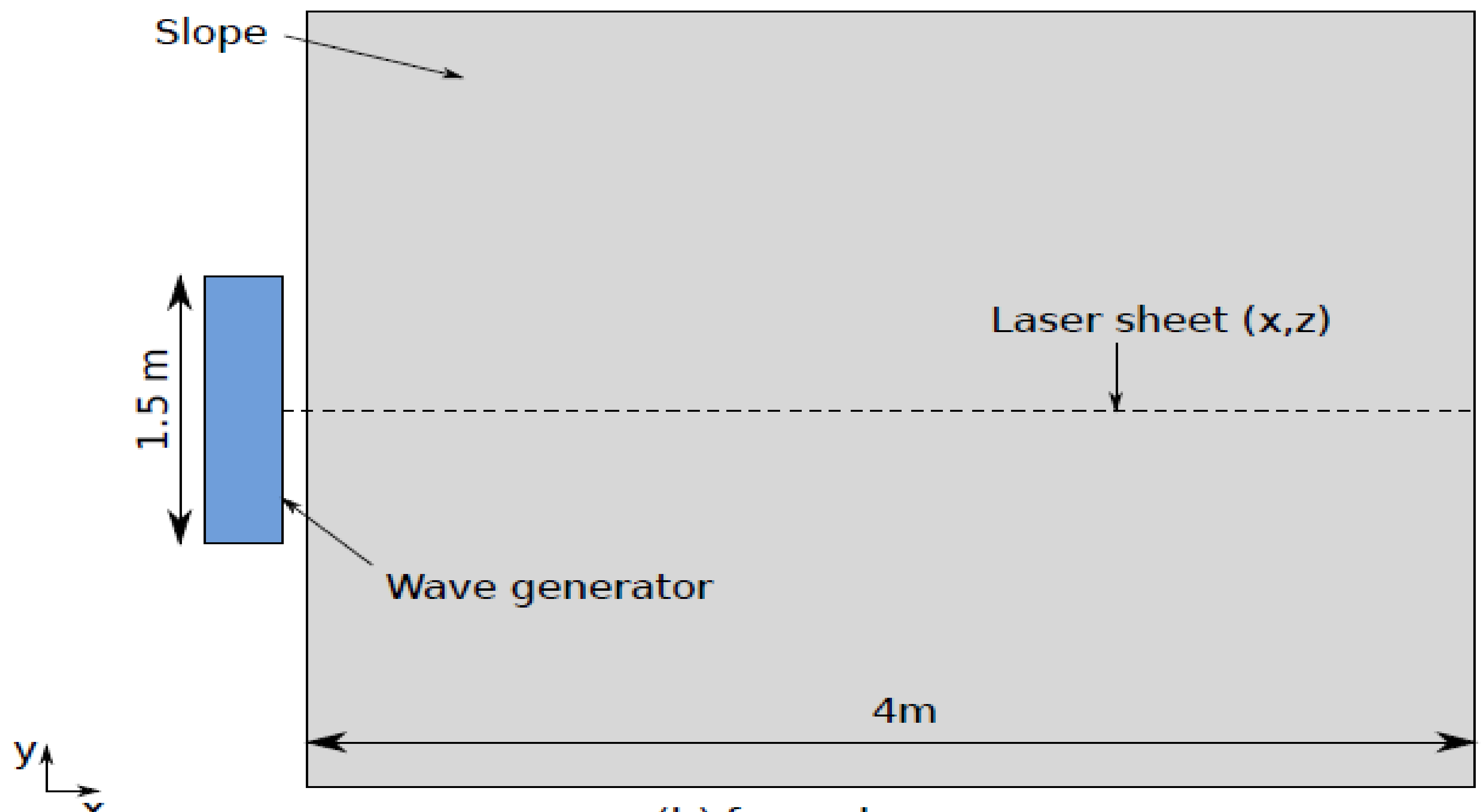


Nonlinear reflexion of internal gravity waves on a uniform slope

Sketch of the laboratory experiment



(a) from the side



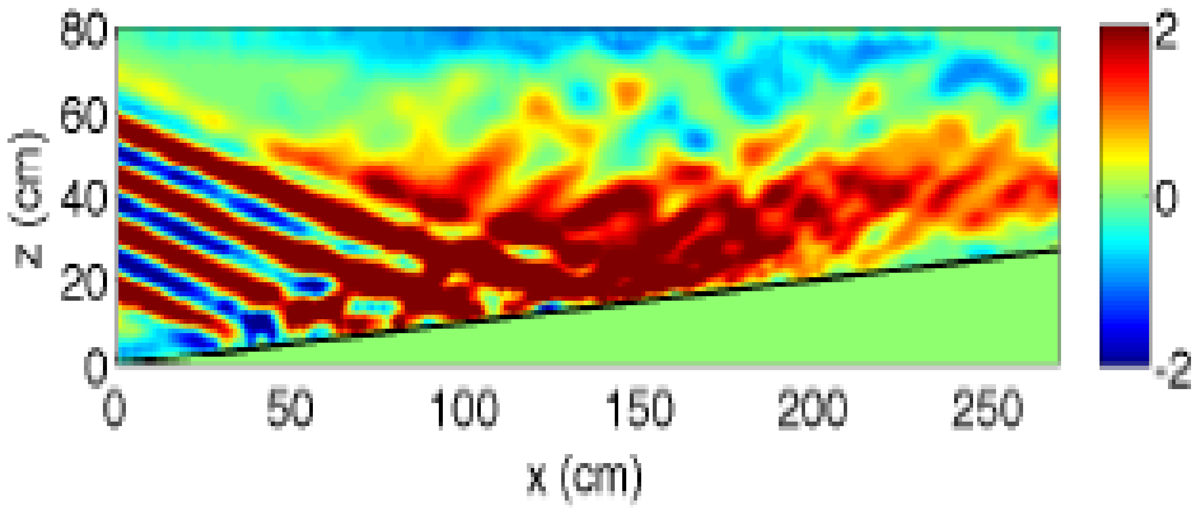
(b) from above

Nonlinear reflexion of internal gravity waves on a uniform slope

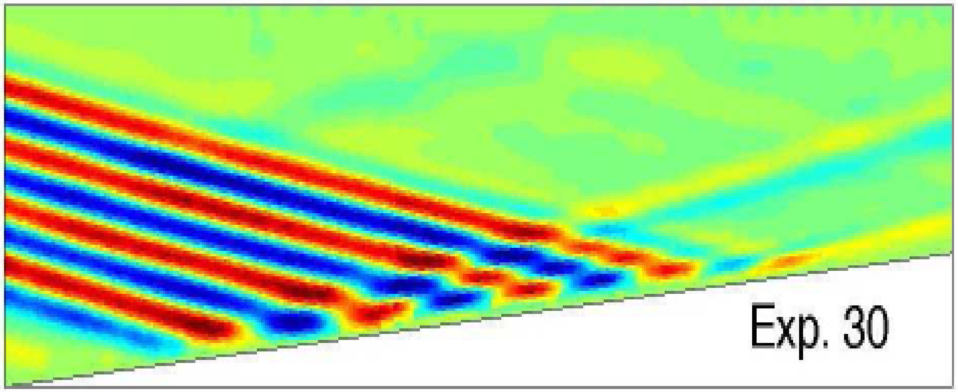
View of the laboratory experiment

Nonlinear reflexion of internal gravity waves on a uniform slope

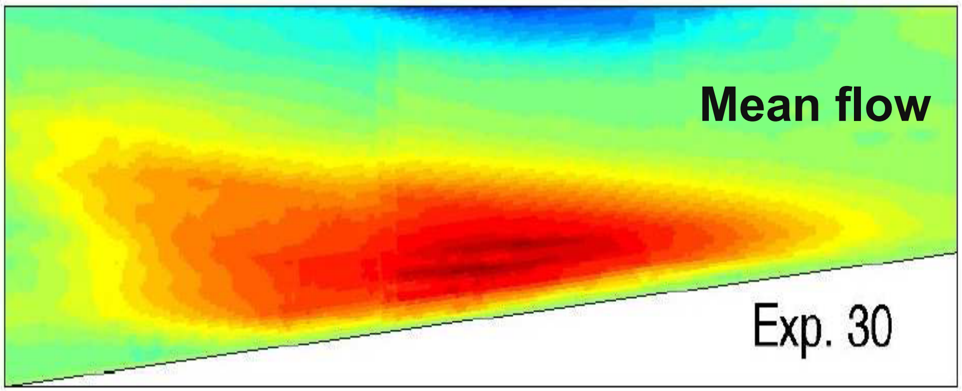
Horizontal velocity component in a vertical plane



Horizontal velocity component (unfiltered)



Horizontal velociy component (filtered at forcing frequency)



Horizontal velocity component (temporal average)

N. Grisouard, PhD , 2010

Nonlinear reflexion of internal gravity waves on a uniform slope

3D numerical simulations (set-up)

NHOES code (Hidenori Aiki)

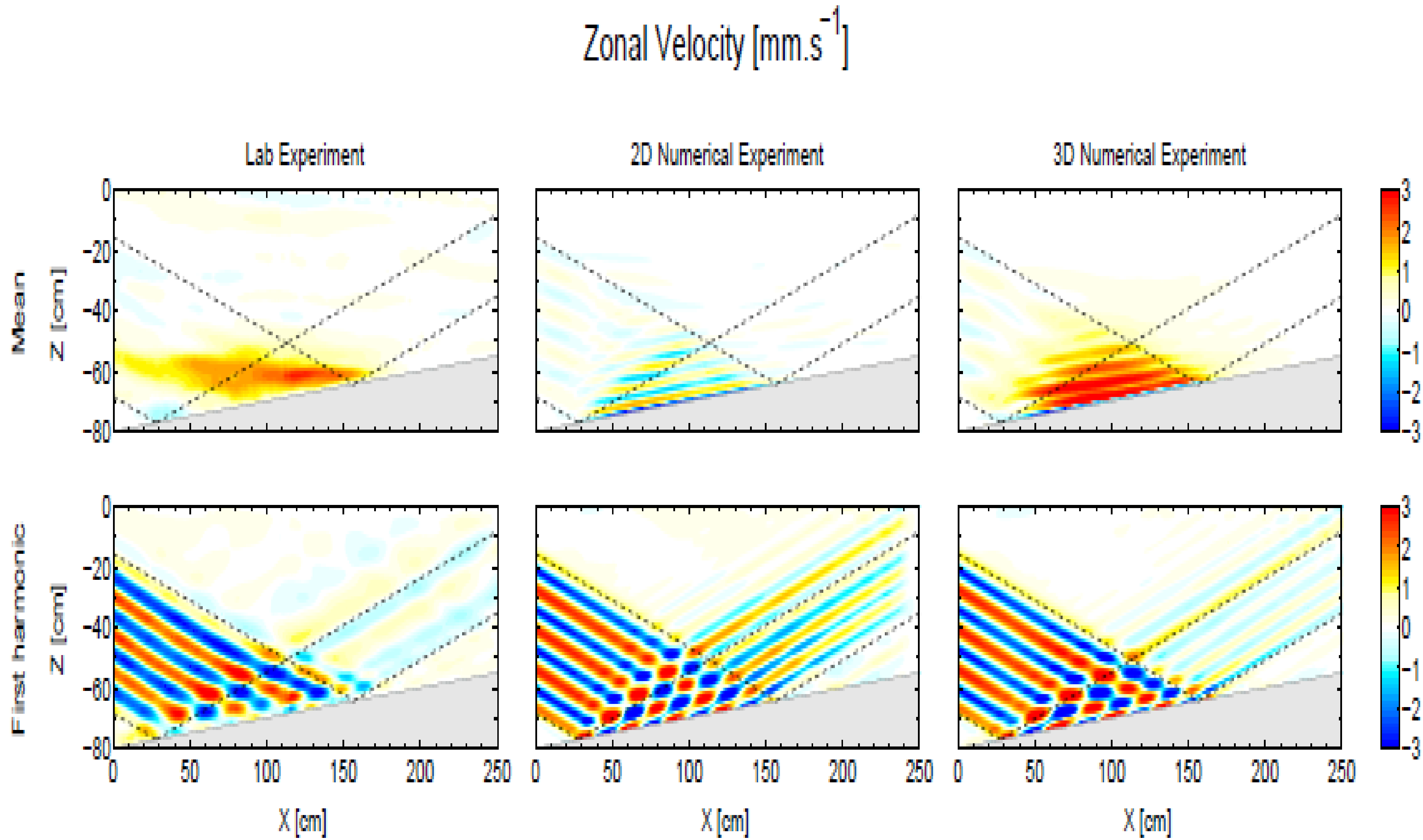
- z-coordinate with partial steps
- non hydrostatic
- free surface
- second order schemes in space and time

Numerical settings: 2D (x, z) and 3D

- Implicit free surface
- Parallelepiped domain:
  - 2D: $L \times h = 2.56\ \mathrm{m} \times 0.8\ \mathrm{m}$
  - 3D: $L \times l \times h = 2.56\ \mathrm{m} \times 2.56\ \mathrm{m} \times 0.8\ \mathrm{m}$
- Resolution:
  - 2D: $\Delta x \times \Delta z = 1\ \mathrm{cm} \times 0.5\ \mathrm{cm}$
  - 3D: $\Delta x \times \Delta y \times \Delta z = 1\ \mathrm{cm} \times 1\ \mathrm{cm} \times 0.5\ \mathrm{cm}$
- Forcing on the *west* side = analytical incident wave: prescribe u and ρ
- linear EOS
- molecular viscosity & diffusivity
- Sponge layer at *east*, *north* and *south*
- Free slip bottom boundary condition

Nonlinear reflexion of internal gravity waves on a uniform slope

Comparison of 3D numerical simulations with laboratory experiments



Conclusion

I. Propagation of internal gravity waves in a non uniform stratification

Interaction of an internal tide beam with a seasonal thermocline
→ generation of solitary waves, as observed in the Bay of Biscay.

II. Propagation of an internal gravity waves in a vertical shear flow

Interaction of lee waves with a critical level → the waves decelerate the wind which generates them.

III. Mean flow induced by an internal gravity wave field

Nonlinear reflexion of an internal gravity wave on a uniform slope
→ strong horizontal mean flow induced, of comparable velocity Amplitude with the incident wave.