On the weak solutions to the Fluid/Rigid Body interaction problem

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Abstract

Consider a rigid body immersed in a fluid which exerce on it contact forces modelized by the cauchy stress tensor, while the velocity field of the fluid obey to Navier-Stokes equations. The total system is contained in a bounded domain Ω of $\mathbb{R}^3$ or $\mathbb{R}^2$. Since the body is moving as time evolve, the domain in which we solve the Navier-Stokes equations is function of the time. We denote by $\mathcal{S}(t)$ and $\mathcal{F}(t)$ the sets occupied respectively by the solid and by the fluid.
We are looking for a fluid velocity field u_f, a solid velocity field u_s, sets $\mathcal{F}(t)$,and $\mathcal{S}(t)$ that solve the following system :

$$\begin{cases} \partial_t u_f + u_f \cdot \nabla u_f = \Delta u_f + \nabla p, & \forall (t, x_t), x_t \in \mathcal{F}(t) \\ \nabla \cdot u_f = 0, & \forall (t, x_t), x_t \in \mathcal{F}(t) \\ u_f = 0, & \forall (t, x_t), x_t \in \partial\Omega \\ u_f = u_s, & \forall (t, x_t), x_t \in \partial\mathcal{S}(t) \\ ml'(t) = -\int_{\partial\mathcal{S}(t)} \Sigma(u_f(t, x_t), p(t, x_t)) \cdot n \\ \frac{d}{dt}\mathcal{J}(t)r(t) = -\int_{\partial\mathcal{S}(t)} (\Sigma(u_f(t, x_t), p(t, x_t)) \cdot n) \wedge (x_t - h_t) \end{cases} \tag{1}$$

The last two equations describe the movement of the solid, and are nothing but Newton second law on the mass center of the solid and angular momentum conservation law of the solid. $h(t)$ is the solid's mass center position at time t, $l(t)$ denotes its mass center velocity at time t, $r(t)$ its angular velocity vector and $\mathcal{J}(t)$ its inertia matrix at time t. $\Sigma(u, p) := -pId + (\nabla u) + (\nabla u)^T$ is the Cauchy stress tensor.

Here the system has been prescribed Dirichlet boundary conditions also known as "no slip conditions", other boundary conditions can be

asked such as Navier boundary conditions ("slip conditions") which seem to lead to more physically acceptable solutions of this system.
Weak solutions to this system with both boundary conditions have been defined and constructed in [1] [2] [3].
For Dirichlet boundary conditions in 2 dimensions, uniqueness for theses weak solution up to collision of the solid with the boundary $\partial\Omega$ have been proved in [4]

References

[1] Hillairet M. ; Topics in the mathematical theory of interactions of incompressible viscous fluid with rigid bodies.

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[3] Necasova S. Chemetov N. ; The motion of the rigid body in viscous fluid including collisions. Global solvability result.

[4] Glass O. Sueur F.; Uniqueness results for weak solutions of two-dimensional fluid-solid systems.