Existence, uniqueness and approximation of the diffusive Peterlin viscoelastic model

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Abstract

The aim of this contribution is to present the existence and uniqueness result for the diffusive Peterlin viscoelastic model describing the unsteady behavior of some incompressible polymeric fluids. The polymers are treated as two beads connected by a spring. For the nonlinear spring force, $\mathbf{F}(\mathbf{R}) = \gamma(|\mathbf{R}|^2)\mathbf{R}$, where $\mathbf{R}$ is the vector connecting the beads, it is not possible to obtain a closed system of equations for the conformation tensor, except by approximating the force law. The Peterlin approximation replaces this law by $\mathbf{F}(\mathbf{R}) = \gamma(\langle|\mathbf{R}|^2\rangle)\mathbf{R}$. That means, the length of the spring in the spring constant γ is replaced by the length of the average spring $\langle|\mathbf{R}|^2\rangle = \text{tr } \mathbf{C}$. Consequently, we can derive the equation for the conformation tensor $\mathbf{C}$, which is in a closed form, see [4]. The later is the time evolution equation with spatial diffusion of the conformation tensor. Using the energy estimates we prove global in time existence of a weak solution to this problem. For more regular data we are also able to show the regularity and consequently the uniqueness of weak solution in two space dimensions. In three space dimensions we have to assume small data in order to obtain the uniqueness result, cf. [1]. We propose a pressure-stabilized characteristics finite element scheme for the approximation of the diffusive Oseen-Peterlin model, cf. [2, 3]. Finally we will show that using $P_1/P_1/P_1$ finite element approximation for the velocity, pressure and the conformation tensor we obtain the first order error estimates, cf. [2].

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Keywords: viscoelastic, existence, uniqueness, finite element, method of characteristics, error estimates

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