

# On uniqueness of symmetric Navier-Stokes flows around a body in the plane

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## Abstract

The two-dimensional exterior problem for the stationary Navier-Stokes equation is known as a difficult problem. Owing to the peculiar difficulties such as the Stokes paradox, the general theory for the non-linear problem is not established yet. Thus far, the existence results of a weak solution tending to zero at infinity are obtained only under suitable symmetry assumptions ([1, 3, 2]).

In this contribution we will investigate the uniqueness of weak solutions, which are less symmetric than those in [1, 3, 2]. Given two symmetric weak solutions  $u$  and  $v$ , we will show that if  $u$  satisfies the energy inequality and  $v$  is sufficiently small in some weighted  $L^\infty$  space, then  $u = v$ . The proof relies upon a density property for the solenoidal vector field and the Hardy inequality for symmetric functions.

**Keywords:** Steady Navier-Stokes flow, Plane exterior domain, Symmetry.

## References

- [1] G. P. Galdi, *Stationary Navier-Stokes problem in a two-dimensional exterior domain*, Handbook of Differential Equations, Stationary partial differential equations, Vol. I., M. Chipot and P. Quittner, eds., North-Holland, Amsterdam, 2004, pp. 71-155.

- [2] K. Pileckas, R. Russo, *On the existence of vanishing at infinity symmetric solutions to the plane stationary exterior Navier-Stokes problem*, Math. Ann. **352** (2012), 643-658.
  
- [3] M. Yamazaki, *Unique existence of stationary solutions to the two-dimensional Navier-Stokes equations on exterior domains*, Mathematical Analysis on the Navier-Stokes Equations and Related Topics, Past and Future-In memory of Professor Tetsuro Miyakawa, Gakuto International Series in Mathematical Sciences and Applications, Vol. **35**, Gakkōtoshō, Tokyo, 2011, pp. 220-241.