Maximal L^p-Regularity of the Spatially Periodic Stokes Operator

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Abstract

We investigate Navier-Stokes flows in the whole space $\mathbb{R}^n$, $n \geq 3$, subject to a spatial periodicity in one direction. To this end we prove maximal L^p-regularity in $L^q(G)$-spaces of the abstract Stokes operator $\mathcal{A}_{G,q}$ in the locally compact abelian group $G := \mathbb{R}^{n-1} \times T$, where T is the one-dimensional torus and the abstract Stokes operator is defined in an obvious way on $L^q_\sigma(G)$. This is achieved by using abstract harmonic analysis in order to establish a concept of the class of Muckenhoupt weights on the group G. With the Muckenhoupt weights at hand we prove an extrapolation theorem for groups, corresponding to the classical extrapolation theorem due to García-Cuerva and Rubio de Francia (1985). This enables us to show $\mathcal{R}$-boundedness of the operator family $\lambda(\lambda - \mathcal{A}_{G,q})^{-1}$ in the weighted space $L^q_{\omega,\sigma}(G)$ of solenoidal vector functions.

As an application we obtain local-in-time existence of spatially periodic solutions to a quasilinear parabolic system describing the dynamics of nematic liquid crystal flows. The underlying model is a simplified Ericksen-Leslie model coupling the Navier-Stokes equation for the velocity with a diffusion equation for the macroscopic orientation d of a molecule.

References

[1] J. Sauer, *Maximal Regularity of the Spatially Periodic Stokes Operator and Application to Nematic Liquid Crystal Flows*, Funkcialaj Ekvacioj, submitted 2014.