Flow Reconstruction from MRV Measurements

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Abstract

We consider a steady flow of an incompressible fluid through a heat exchanger covering a domain Ω with boundary $\partial\Omega$ governed by the Navier-Stokes equations

$$\begin{aligned} -\nu\Delta u + u\cdot\nabla u + \nabla p &= 0 \quad \text{in } \Omega \\ \operatorname{div} u &= 0 \quad \text{in } \Omega \\ u &= g \quad \text{on } \partial\Omega. \end{aligned} \tag{NS}$$

Here ν denotes the kinematic viscosity, u the velocity field, p the kinematic pressure in the fluid, and g an unknown velocity profile on the boundary. These equations are fairly well understood from a theoretical point of view [2].

Our goal is to identify the unknown velocity profile g at the boundary by measurements of the velocity field u in a subdomain $\Omega_d \subset \Omega$, which is a typical inverse problem [3]. The measurements are given by means of noisy MRV[1] measurements [1].

After regularization the inverse problem can be formulated as an optimal control problem of the form

$$\begin{aligned} &\min \|u - u^\delta\|^2_{L^2(\Omega_d)} + \alpha|g - \bar{g}|^2 \\ &\text{s.t.} \quad \text{(NS)} \quad \text{holds.} \end{aligned}$$

Hereby u^δ denotes the measurements, $\alpha > 0$ a constant, and $\bar{g}$ an expected boundary profile, e.g. $\bar{g} = 0$ on walls and a Poiseuille inflow profile. The norm $|\cdot|$ has to be chosen appropriately in order to guarantee the existence of a minimizer. Such problems are referred

[1] magnetic resonance velocimetry

to as pde-constrained optimization and are studied extensively in the literature. We refer to [4] and [5] and the references given therein.

We establish the existence of minimizers and stability results concerning the choice of α and the size of the measurement error. We also discuss the computation of quantities of interest, e.g. the pressure drop or the total mass flow along the channel.

Various generalizations are possible: Besides the problem considered above also some material parameters, the viscosity or parameters in the boundary condition can be identified. Time- and temperature-dependent problems can be considered. Since the measurements are taken at high Reynolds numbers also turbulence models are suitable. In each case reasonable numerical implementations have to be studied as well.

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Keywords: flow identification, incompressible flow, stationary flow

References

[1] Wassermann, F., Grundmann S.: Flow visualization of a staggered pin fin array using Magnetic Resonance Velocimetry, 15th ERCOFTAC-SIG15/IAHR Workshop (2011)

[2] Temam, R.: Navier-Stokes Equations - Theory and Numerical Analysis, American Mathematical Society (2001)

[3] Engl, H. W., Hanke, M., Neubauer, A.: Regularization of Inverse Problems. Kluwer Academic Publishers (2000)

[4] Tröltzsch, F., Wachsmuth, D.: Second-order sufficient optimality conditions for the optimal control of Navier-Stokes equations. ESAIM COCV 12, 93–119 (2006)

[5] Gunzburger, M., Hou L., Svobodny T.: Analysis and finite element approximation of optimal control problems for the stationary Navier-Stokes equations with distributed and Neumann controls. Math. Comp. 195, 123–151 (1991)