Time-dependent singularities in the Navier-Stokes system

Xiaoxin Zheng

xiaoxingyah@163.com

Abstract

The Cauchy problem for the incompressible Navier–Stokes system in the whole three dimensional space reads

\[ \partial_t u + (u \cdot \nabla)u - \Delta u + \nabla p = 0, \quad (x, t) \in \mathbb{R}^3 \times (0, \infty), \]
\[ \text{div} u = 0. \]

We have proved that, for a given Hölder continuous curve in \{\( (\gamma(t), t) \in \mathbb{R}^3 \times (0, \infty) : t > 0 \)\}, there exists a solution to this system which is smooth outside this curve and singular on it. This is a pointwise solution of the system outside the curve, however, as a distributional solution on \( \mathbb{R}^3 \times (0, \infty) \), it solves an analogous Navier-Stokes system with a singular force concentrated on the curve.

This is a joint work with Grzegorz Karch.

Keywords: Navier–Stokes system, incompressible fluid, time-dependent singularity, Slezkin-Landau solutions.