The Role of the Pressure in Finite Element Methods for Incompressible Flow Problems

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Lecture 1:

On the choice of finite element spaces for velocity and pressure

The discrete inf-sup condition is introduced. It is shown with examples that some low order pairs of spaces do not satisfy this condition. Some ways for proving the discrete inf-sup are outlined and pairs of spaces are given that satisfy this condition.

Lecture 2:

Finite element error analysis for inf-sup stable pairs of finite element spaces

The standard finite element error analysis for the Stokes equations is presented. Error bounds for velocity and pressure are derived. The impact of the discrete pressure on the error bounds for the velocity is emphasized.

Lecture 3:

Reducing the impact of the finite element pressure on the finite element velocity error

Some techniques for reducing or even eliminating the impact of the discrete pressure in the error bound for the velocity are presented. These techniques include the grad-div stabilization, the construction of weakly divergence-free pairs of finite element spaces, and the use of appropriately reconstructed test functions (so-called pressure-robust finite element methods).

Lecture 4:

Finite element methods that circumvent the discrete inf-sup condition

There are several proposals for circumventing the discrete inf-sup condition. These methods allow in particular the use of equal-order pairs of finite element spaces. Some of these proposals will be presented, with the emphasis on the pressure-stabilized Petrov-Galerkin (PSPG) method.