Error estimates for two types of Lagrange-Galerkin scheme for the Peterlin viscoelastic model

M. Lukáčová, H. Mizera, H. Notsu, M. Tabata

Mathematical model

The Peterlin viscoelastic model [P]

\[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \beta(\nabla u + (\nabla u)^T) - \beta \nabla \phi, \quad \nabla : u = 0 \]

\[ \frac{\partial \phi}{\partial t} + (u \cdot \nabla) \phi = \chi(\nabla \cdot (\nabla \phi)) - \phi (\nabla \cdot (\nabla \phi)) + \epsilon \Delta \phi \]

on \( \Omega \times (0, T) \)

\[ (u(0, \cdot), \phi(0, \cdot)) = (u_0, \phi_0) \] in \( \Omega \).

\( \Omega \subset \mathbb{R}^{d} \) bounded smooth domain, \( \mu \) fluid viscosity, \( \epsilon \) elastic stress diffusivity

Existence of global weak solutions

Let \( d = 2, 3 \) and let \( (u_0, \phi_0) \in L^2(\Omega) \times L^2(\Omega) \).

There exists a weak solution to \([P]\) such that

\[ u \in L^\infty(0, T; L^2(\Omega)) \cap L^2(0, T; H^1_0(\Omega)), \quad \phi \in L^2((0, T) \times \Omega) \cap L^{1+\delta}(0, T; W^{1,1+\delta}(\Omega)), \]

\( d = 2 \) \( [4] \)

\( d = 3 \) \( [4] \)

\( d = 3 \) \( \mathbb{R}^d \) pressure, \( C \in \mathbb{R}^{d \times d} \) constant tensor

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Numerical approximation of the Oseen-type Peterlin viscoelastic model by the stabilized Lagrange-Galerkin method

- \( \psi(s) = \phi(s) \) and \( \phi(s) = s \)
- Conforming finite element approximation of \( u, p \) and \( \phi \) by continuous piecewise linear functions + pressure stabilization
- Material derivative discretized by the method of characteristics

Numerical experiments

Experimental order of convergence

Convergence domain: \( (x, y, t) \in (0,1)^2 \times (0,0.5) \)

The Oseen-type Peterlin viscoelastic model

The convective terms in \([P]\) are linearized by a given velocity \( w : \Omega \times (0, T) \to \mathbb{R}^d \).

References


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\[ \psi(s) = \phi(s) = s \]

Then, for more regular data, there exists a unique global weak solution to \([P]\) with,\( \psi(s) = \phi(s) = s^2 \).

\[ \psi(s) = \phi(s) = s \]