Potential theory for the time–dependent linear equations of incompressible Stokes and Oseen fluid flows

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Abstract

The thrust of this study is to carefully evaluate the consequences of the mathematical formulations of Stokes and Oseeen for slow incompressible flow systems using potential methods with new uniquely formulated potential kernels. The Stokes flow regime has been studied in great detail for many years, much of it summarized in the book by Ladyzhenskaya. The incompressible flow field descriptions proposed by Oseen has not had the same attention.

The relationship of these mathematical formulations for slow incompressible flow to the Navier-Stokes equations is reviewed. The scaling and linearization necessary to suggest the Oseen/Stokes linearized system for the flow is also quickly reviewed. The free space kernel functions in the fundamental solution matrix for the velocity field are determined by Fourier Transform methods. The resulting unusual inverse transform problem leads to the new kernel functions determined in terms of the usual heat kernel together with Kummer functions that are not widely familiar but are well documented. The general properties of the integral operators defined by these kernels are investigated with the details depending heavily on the properties of the Kummer functions and the standard heat kernel. The pressure parameter is shown to be tightly related to the incompressibility condition and determined by a classical potential formula.

The Cauchy initial value problem is solved conjointly for both flow regimes with special attention to the uniqueness properties. Boundary

value problems for both time dependent and time independent problems are considered for velocity specified boundary data and force specified data. The solution approach is modeled after the theory of thermal potentials using the new specified kernel functions. The functions in the literature for the stationary Stokes problems are recovered with extension to Oseen's problems. The non-stationary problems are also solved using appropriate time dependent potential functions based on the new kernel functions in similarity with thermo potential theory.

The boundary value potentials lead to vector valued integral equations with unusual matrix kernels whose solutions provide the velocity vector fields from which the final pressure formulas are determined. Special attention is paid to the pressure parameter because of its unique role in this class of problems. A particular half space boundary value problem is completely solved as an example of the methods.

Keywords: Stokes problem, Oseen problem, linearized, potential, kernel.