

On Leray's self-similar solutions of the Navier-Stokes equations

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Abstract

In our talk we will present recent results concerning the local regularity problem for suitable weak solutions to the Navier-Stokes equations. By using the local pressure projection we are able to modify the proof of Caffarelli-Kohn-Nirenberg in [1] to get a new ε regularity condition. Namely, for every $\frac{3}{2} < q \leq 2$ there exists a sufficiently small constant $\varepsilon_q > 0$ such that for any local suitable weak solution $u : Q_T \rightarrow \mathbb{R}^3$ to the Navier-Stokes equation the condition $r^{q-3} \|u\|_{L^\infty(t_0-r^2, t_0; L^q(B_r(x_0)))}^q \leq \varepsilon_q^q$ implies that $u \in L^\infty(Q_{r/2}(x_0, t_0))$, and it holds

$$\|u\|_{L^\infty(Q_{r/2}(x_0, t_0))} \leq C_q r^{-1} \|u\|_{L^\infty(t_0-r^2, t_0; L^q(B_r(x_0)))}$$

for some constant $C_q > 0$. As a consequence we extend Tsai's result [2] on Leray's self-similar solutions to the Navier-Stokes equations to the case that the profile U belongs to $L^q + L^\infty$ for some $\frac{3}{2} < q < 3$.

Keywords: Navier-Stokes equation, local regularity, self similar solutions.

References

- [1] L. Caffarelli, R. Kohn and L. Nirenberg, *Partial Regularity of suitable weak solutions of the Navier-Stokes equations*, Comm. Pure Appl. Math. 35 (1982), 771-831.
- [2] T.-P. Tsai, *On Leray's self-similar solutions of the Navier-Stokes equations satisfying local energy estimates*, Arch. Rational Mech. Anal. 143 (1998), 29-51.