Numerical simulations of flow induced vocal folds vibrations

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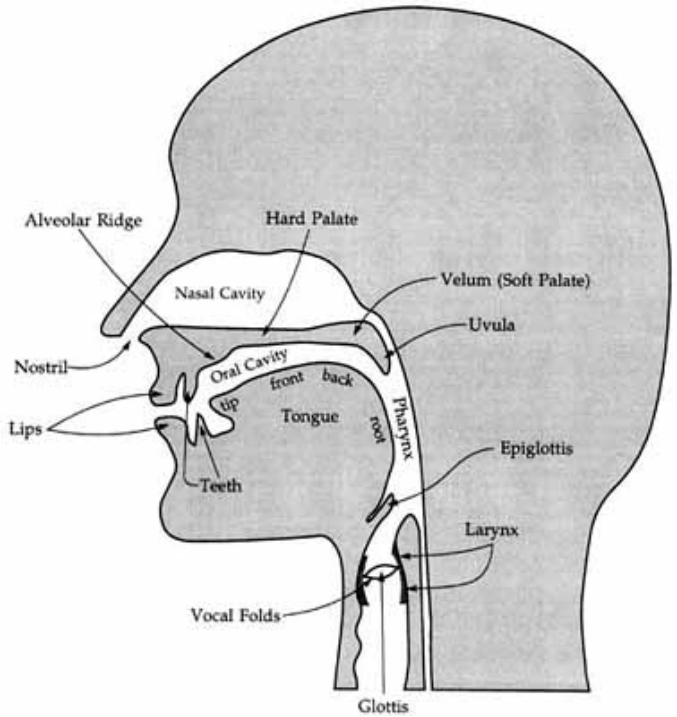
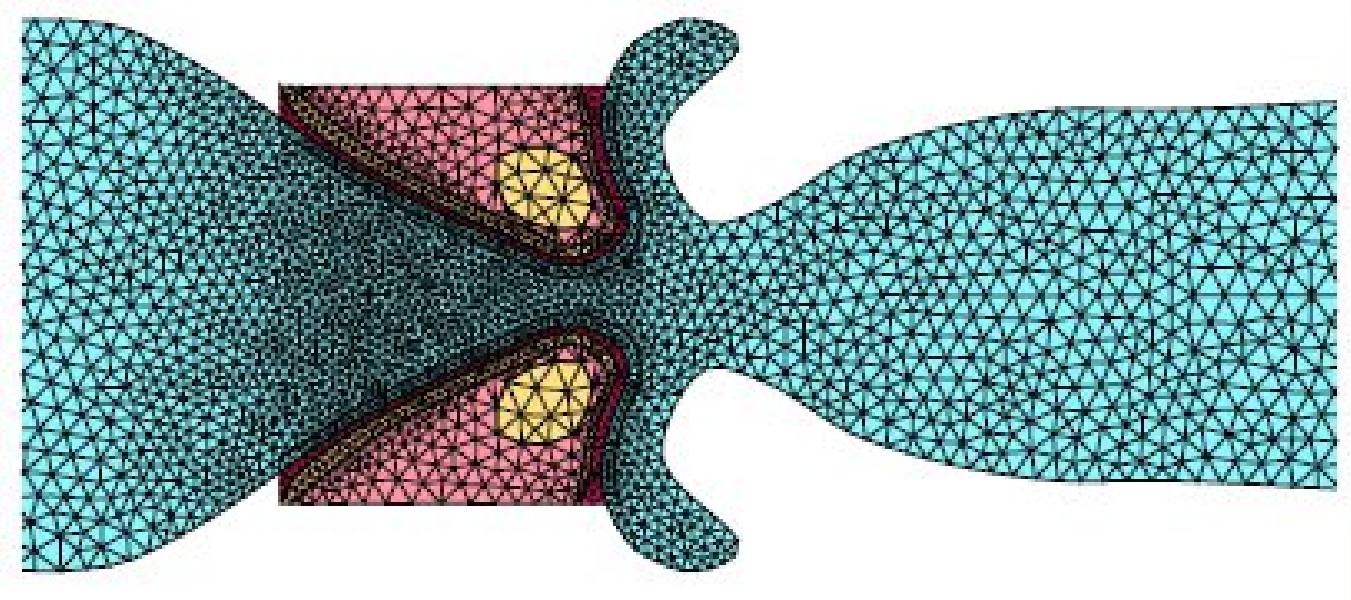
Summer School and Workshop, Fluids Under Pressure, Prague 2016

How the human voice is created?



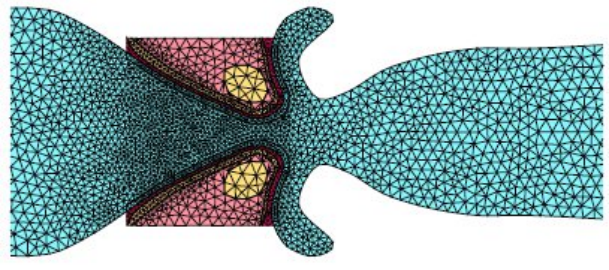
- voice - extremely important for millions of people
- mathematical modelling can lead better understanding,
- challenging: **fluid-structure-acoustic interactions**
- courtesy: A Beginner's Guide to Phonetics, Jean Peccei

Mathematical model/2D



- fluid problem
- structure problem
- interface conditions

Mathematical model/2D



Fluid flow: Navier-Stokes system in ALE form

$$\begin{aligned} D^{\mathcal{A}}\mathbf{v}/Dt + (\mathbf{v} - \mathbf{w}_g)\cdot\nabla\mathbf{v} + \nabla p - \nu\triangle\mathbf{v} &= 0 \\ \nabla\cdot\mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

Structure: Linear elasticity

$$\varrho^b \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \tau_{ij}^b}{\partial x_j} = f_i, \text{ in } \Omega^b, \tag{1}$$

- interface conditions (kinematics/dynamics)
- **difficulties: contact, mesh distorsion, boundary conditions(?)**

Numerical approximation (fluid)

ALE derivative:

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} \approx \frac{3\mathbf{v}_{n+1} - 4\tilde{\mathbf{v}}_n + \tilde{\mathbf{v}}_{n-1}}{2\Delta t}$$

NS - weak reformulation ($\mathbf{b} = (\mathbf{v} - \mathbf{w}_g)$)

$$\left(\frac{3\mathbf{v}}{2\Delta t}, \mathbf{z}\right) + \Big([\mathbf{b}\cdot\nabla]\,\mathbf{v}, \mathbf{z}\Big) + (\nu\nabla\mathbf{v} - p\mathbf{I}, \nabla\mathbf{z}) + (\nabla\cdot\mathbf{v}, q) = \left(\frac{4\tilde{\mathbf{v}}_n - \tilde{\mathbf{v}}_{n-1}}{2\Delta t}, \mathbf{z}\right)$$

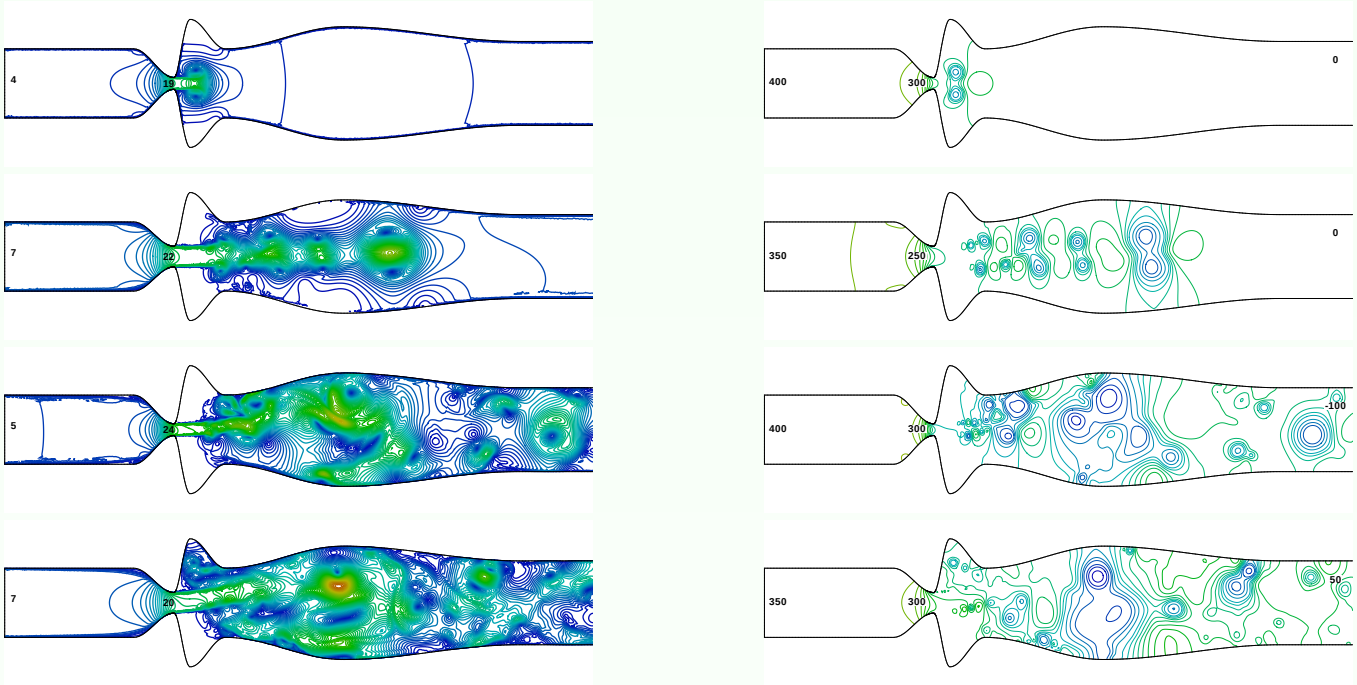
Stabilization (SUPG+PSPG) $\psi = \delta_K\left[(\mathbf{b}\cdot\nabla)\mathbf{z} + \nabla q\right]$:

$$LHS + \sum_K \left(\frac{3\mathbf{v}}{2\Delta t} - \nu\triangle\mathbf{v} + (\mathbf{b}\cdot\nabla)\,\mathbf{v} + \nabla p, \psi\right)_K = \sum_K \left(\frac{4\mathbf{v}^n - \mathbf{v}^{n-1}}{2\Delta t}, \psi\right)_K + RHS$$

Include div-div term (LHS):

$$LHS + \sum_{K\in\tau_h} \tau_K\left(\nabla\cdot\mathbf{u}, \nabla\cdot\mathbf{v}\right)_K$$

Numerical approximation (structure + interface)



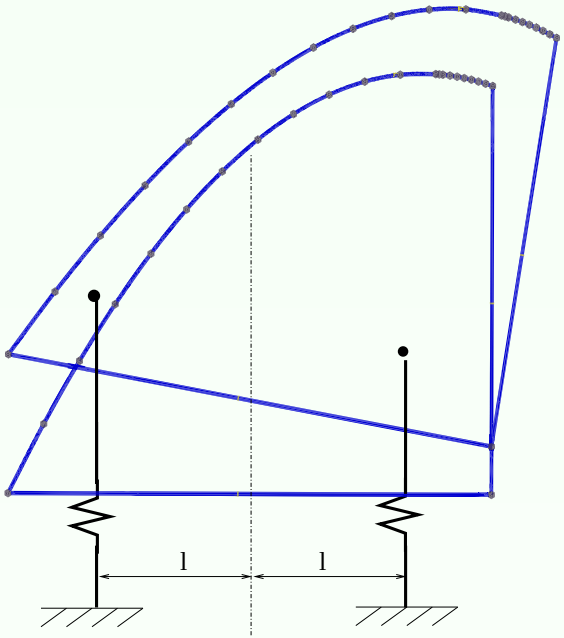
- FE choice: velocity P_2, pressure, P_1
- linearization of stabilized formulation
- coupled with structure - strong coupling
- outflow boundary (!) - modified do-nothing b.c. AVI AVI (detail)

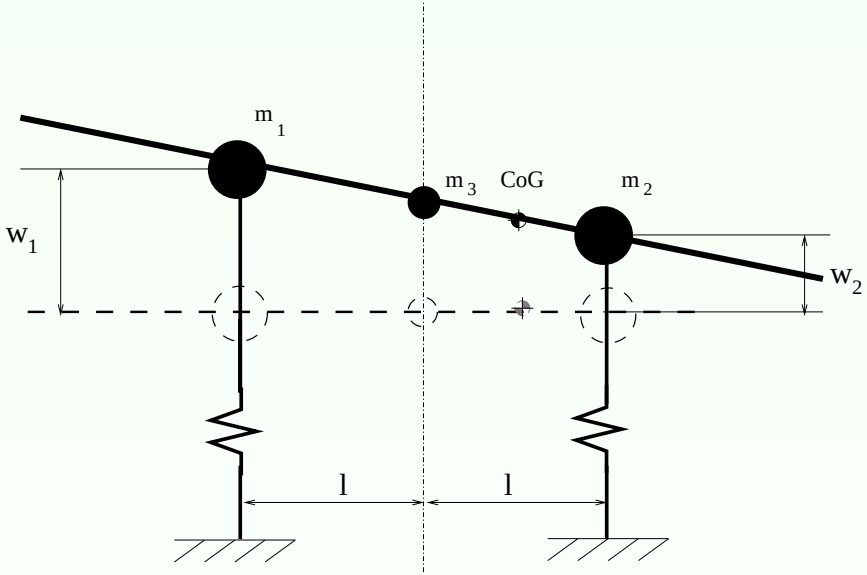
Flow interactions with elastic structure (A. Kosik)

AVI

Use a simplified model!

Elastic structure motion - 2 dof model





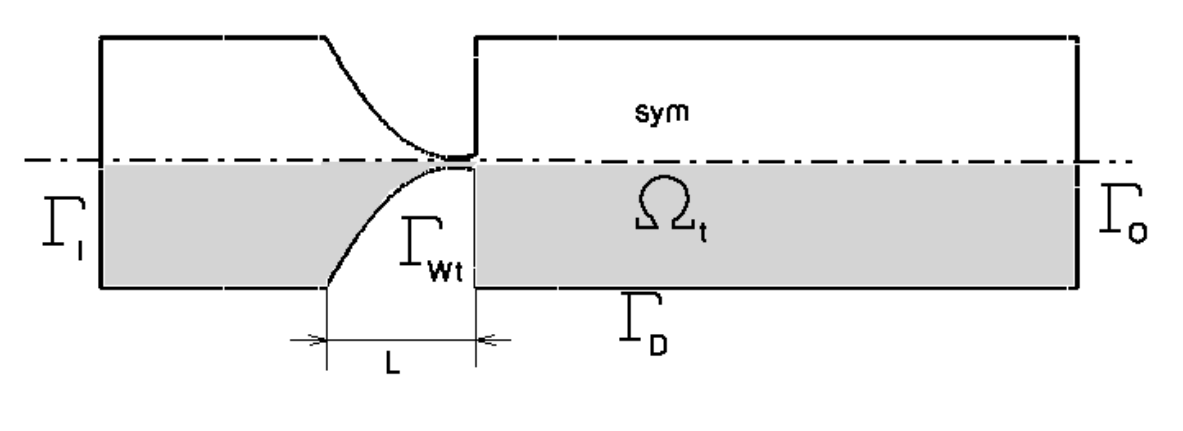
$$\mathbb{M}\begin{pmatrix}\ddot{w}_1\\ \ddot{w}_2\end{pmatrix}+\mathbb{B}\begin{pmatrix}\dot{w}_1\\ \dot{w}_2\end{pmatrix}+\mathbb{K}\begin{pmatrix}w_1\\ w_2\end{pmatrix}+\begin{pmatrix}F_1\\ F_2\end{pmatrix}+\begin{pmatrix}F_1^S\\ F_2^S\end{pmatrix}=0$$

$$\mathbb{M}=\begin{pmatrix}m_1+\frac{m_3}{4} & \frac{m_3}{4}\\ \frac{m_3}{4} & m_2+\frac{m_3}{4}\end{pmatrix},\qquad \mathbb{K}=\begin{pmatrix}c_1 & 0\\ 0 & c_2\end{pmatrix}$$

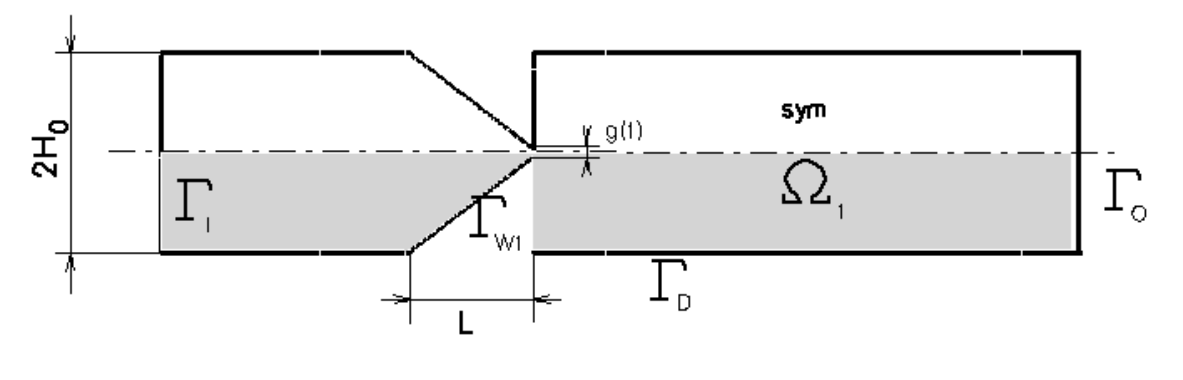
See Horáček, Šidlof, Švec 2002.

Coupled FSI problem

Numerical approximation of flow induced vibrations



Model M



Model F

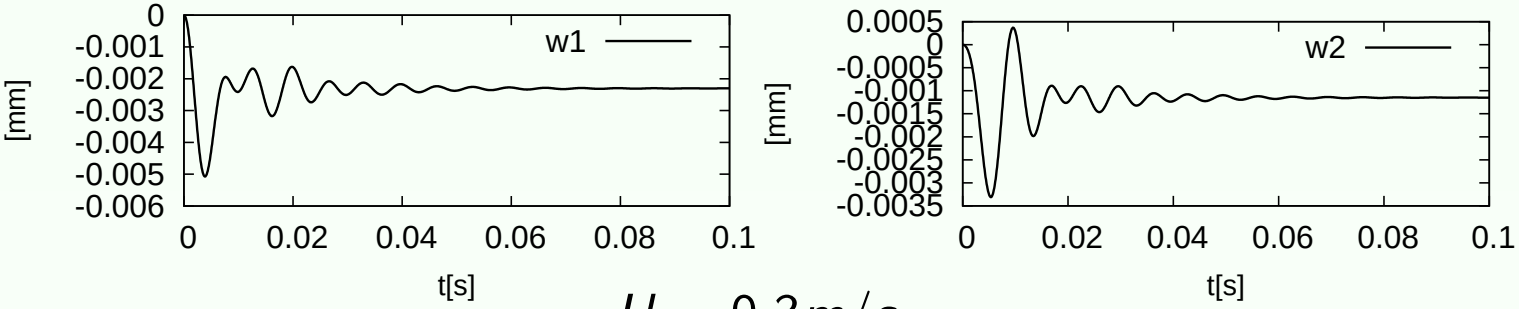
Compare low-fidelity model, (Horacek, Svec, 2002)

	Model F	Model M
shape	$a_f(x)$	$a_m(x)$
m [kg]	3.274×10^{-4}	4.812×10^{-4}
I [kg/m^2]	1.341×10^{-9}	2.351×10^{-9}
e [m]	1.133×10^{-3}	0.771×10^{-3}

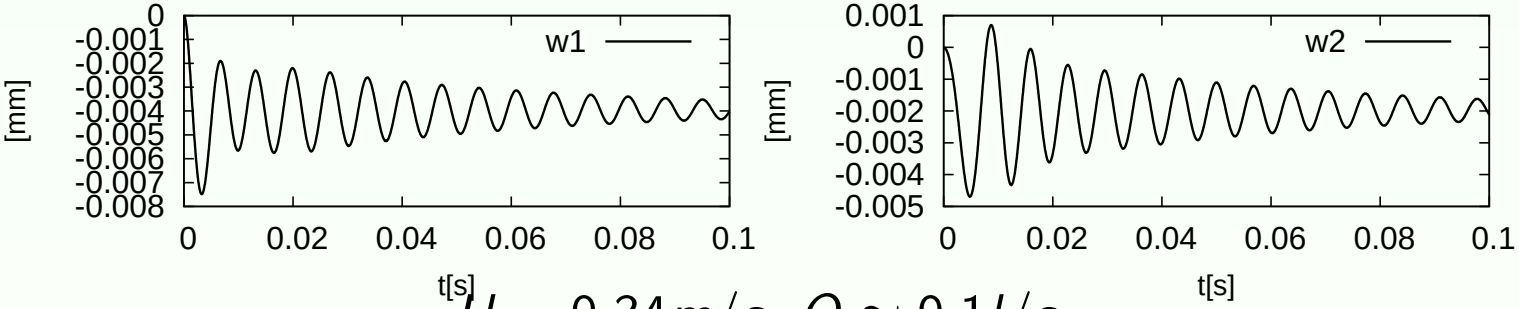
	Model F	Model M
c_1 [N/m]	44.8	56
c_2 [N/m]	84.6	174.3
f_1 [Hz]	100	100
f_2 [Hz]	160	160

Numerical results (Model F)

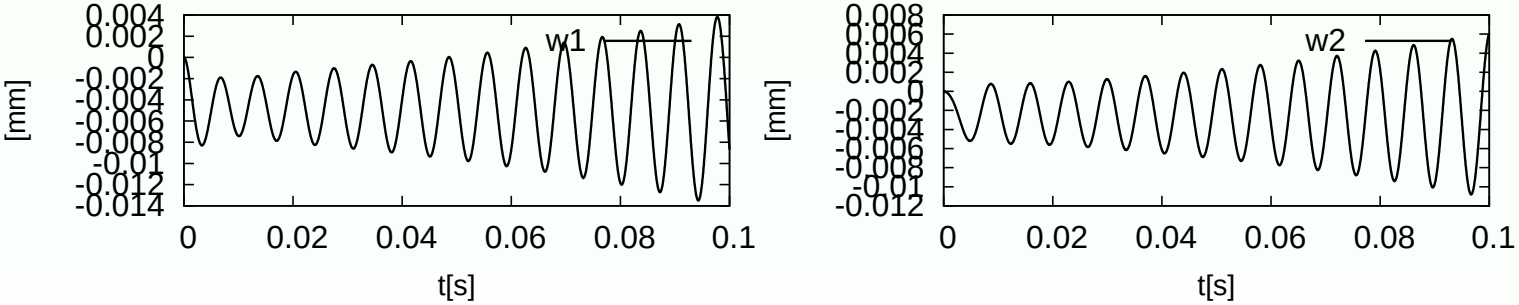
$U = 0.2m/s$



$U = 0.3m/s$

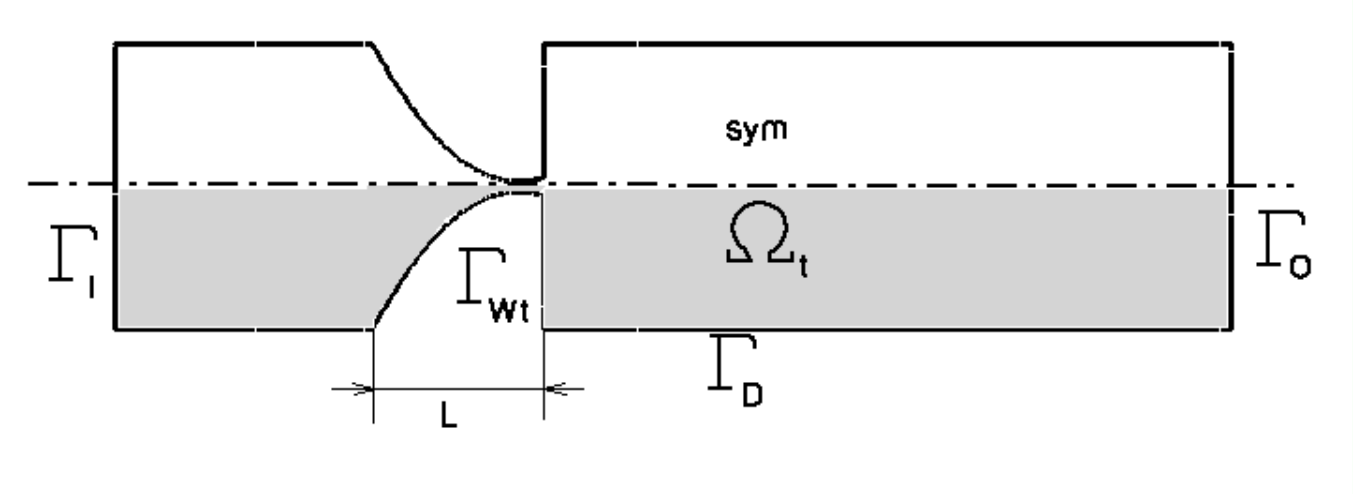


$U = 0.34m/s,\ Q \approx 0.1l/s$



Numerical results

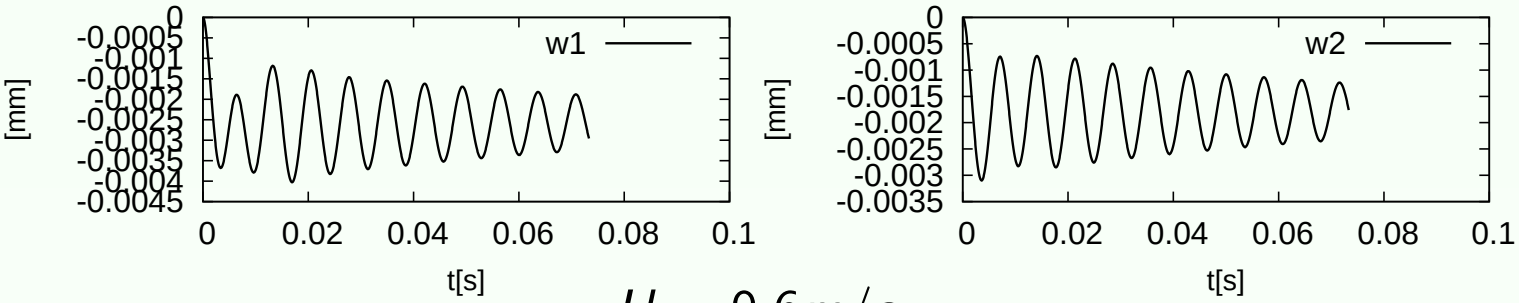
Numerical approximation of flow induced vibrations



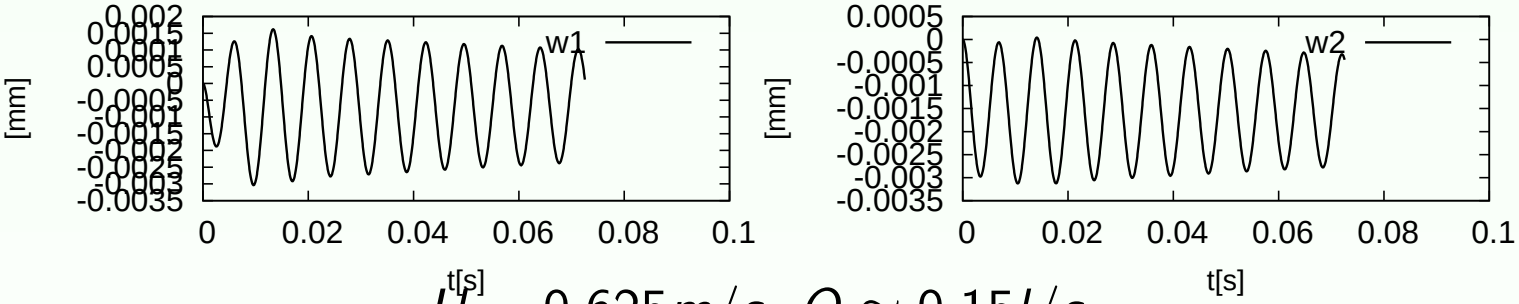
Compare low-fidelity model, (Horacek, Svec, 2002)
Inlet: velocity profile

Numerical results (Model M)

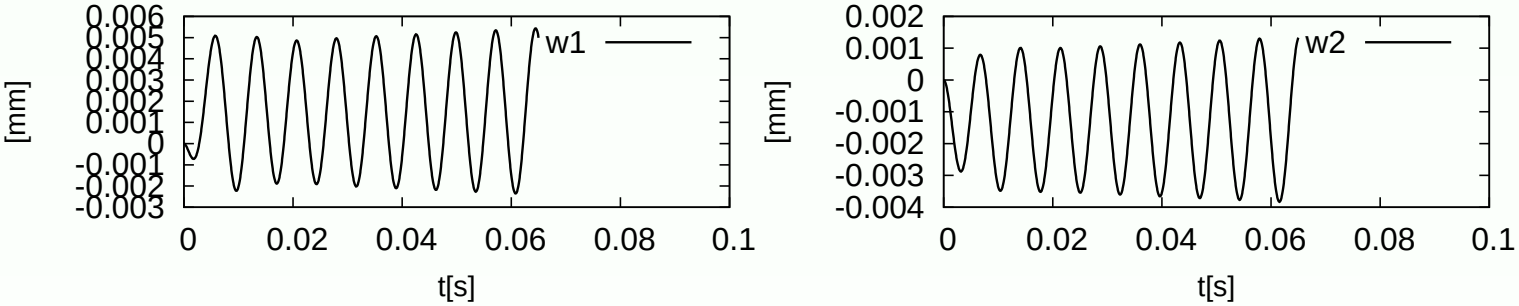
$U = 0.575 m/s$



$U = 0.6 m/s$

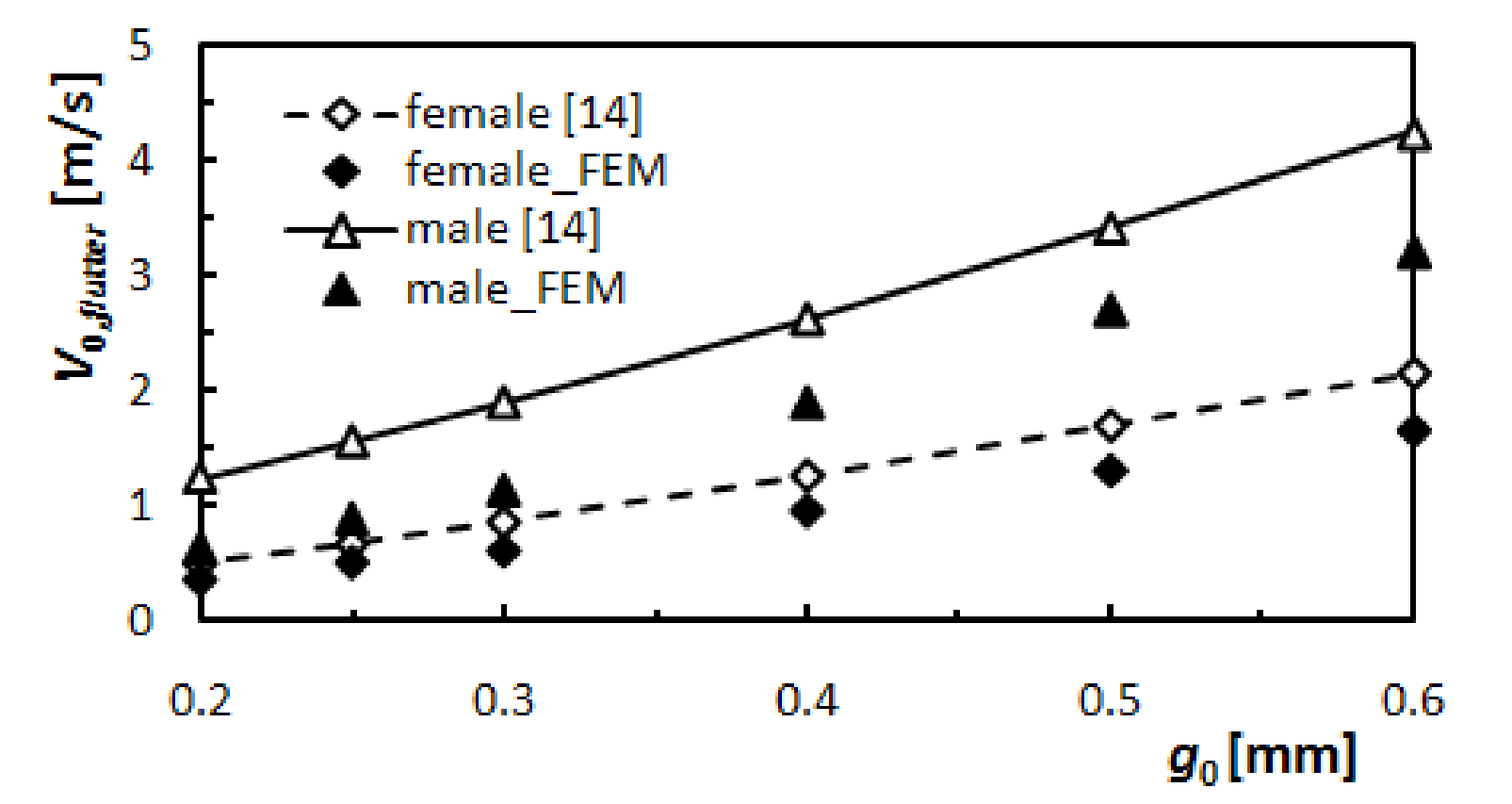


$U = 0.625 m/s,\ Q \approx 0.15 l/s$



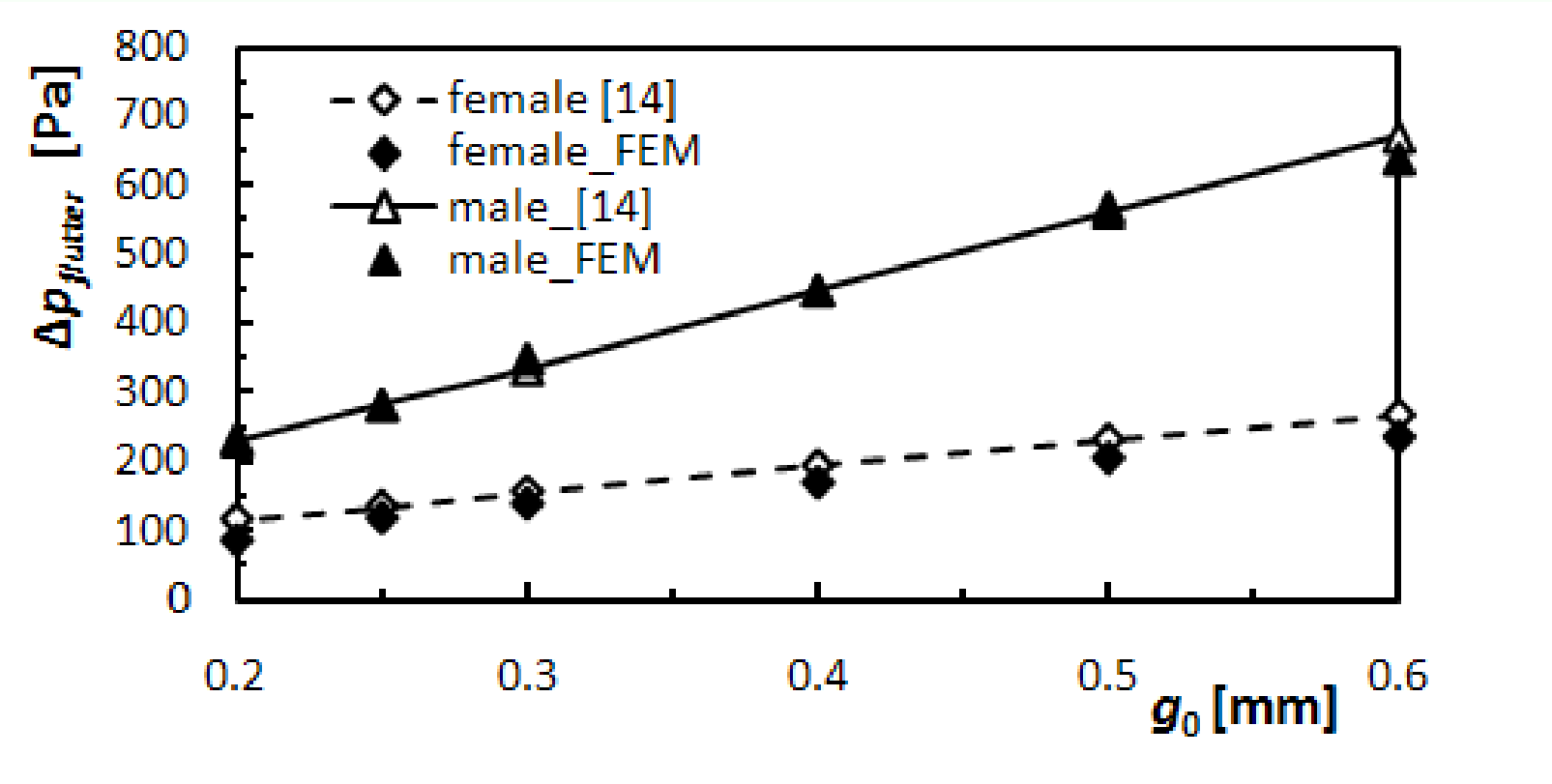
Numerical results

Quantitative comparison: flutter velocity

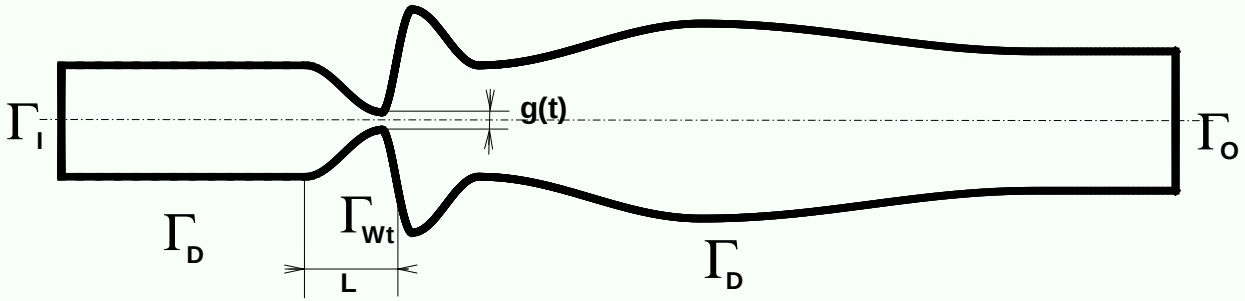


Numerical results

Quantitative comparison: pressure



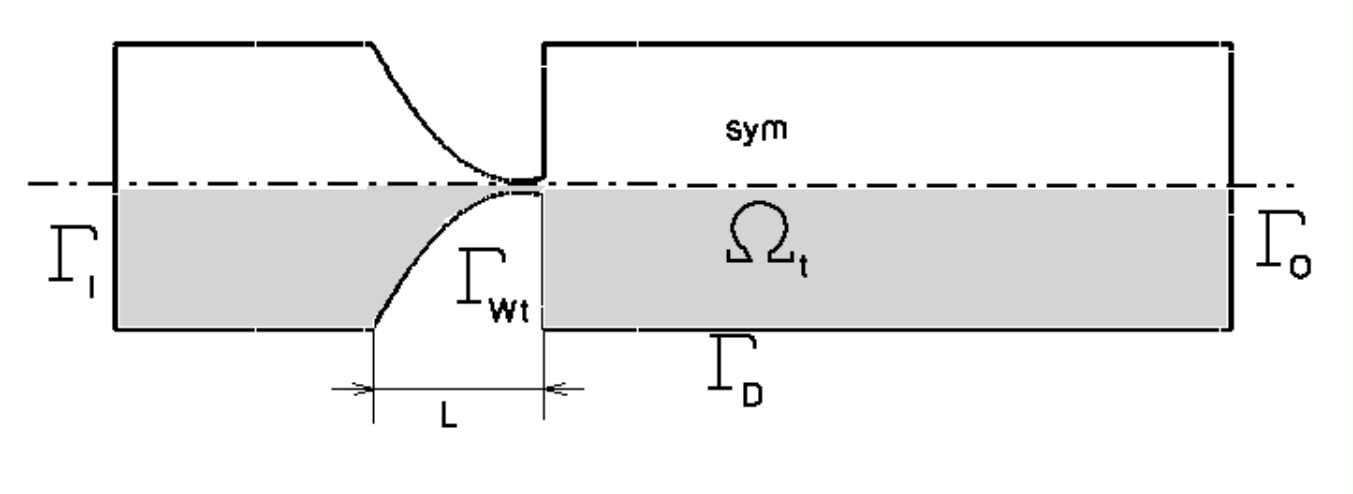
Boundary condition



- **Outlet boundary condition** - modified do-nothing bc
- **Inlet boundary condition**
  - either: flow velocity $(U_0, 0)$
  - or: pressure Δp

Coupled FSI problem

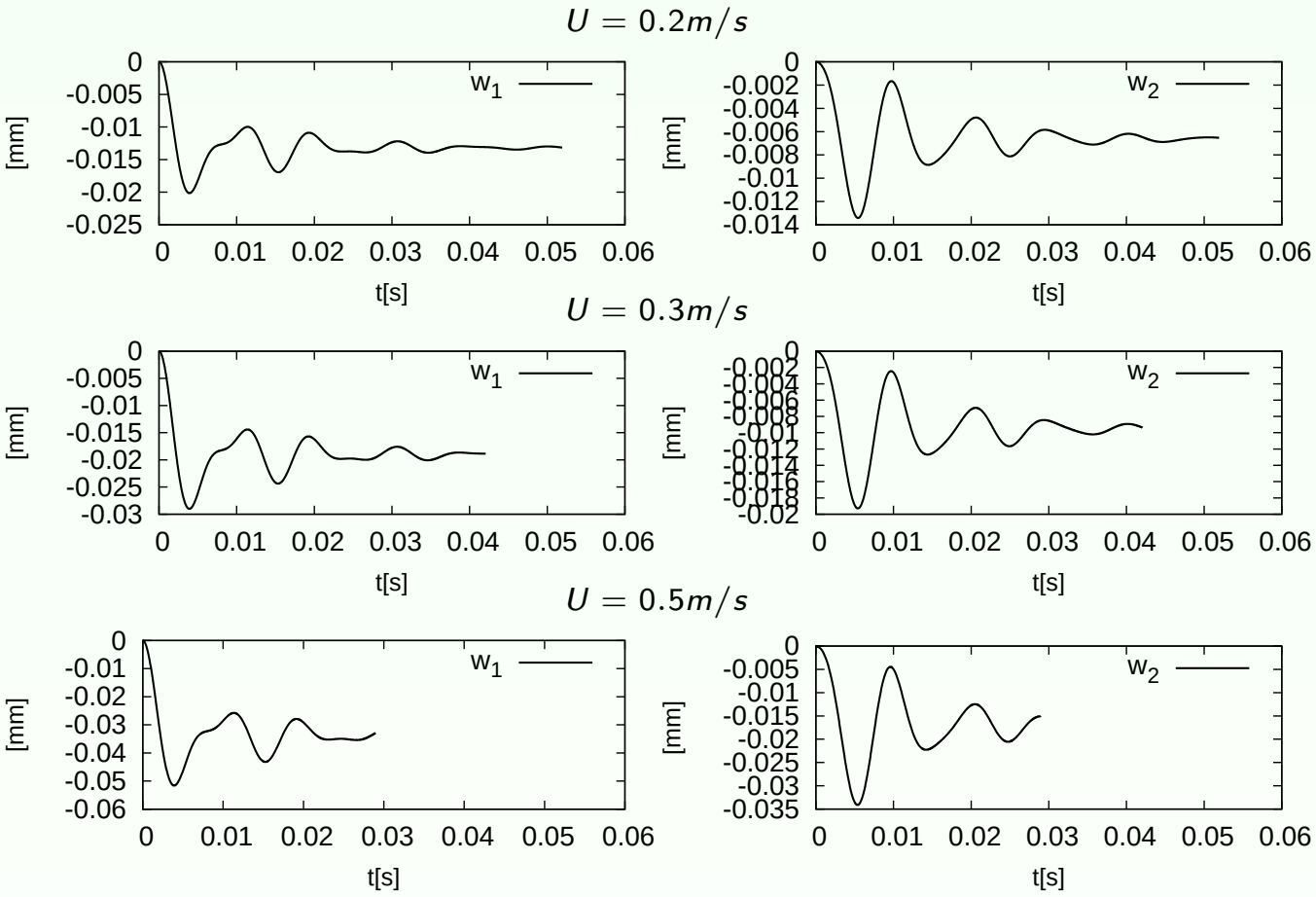
Numerical approximation of flow induced vibrations



Compare low-fidelity model, (Horacek, Svec, 2002)

Inlet: pressure

Numerical results (Model F, pressure drop)



- **Increase of $\Delta p \rightarrow$ ael instability (AVI)**
- **but instability for non-physical values** ($\Delta p = 10kPa$, $U_{max} \approx 180m/s$)
- **low fidelity model: ael instability for** $0.4m/s$

Pressure drop problem

Natural boundary condition - assumed to be more realistic, but it is not ...

- prescribed pressure (velocity varies)
- time dependent - very high variations of inlet velocity
- outlook: use a combination of inlet velocity/inlet pressure b. c.

Conclusion

- Aeroelastic simulations of flow interacting within the vibrating vocal fold was perfomed.
- The qualitative and quantitative results were discussed.
- The choice of appropriate boundary conditions were discussed.