Numerical simulations of flow induced vocal folds vibrations

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How the human voice is created?

- voice - extremely important for millions of people
- mathematical modelling can lead better understanding,
- challenging: **fluid-structure-acoustic interactions**

- courtesy: A Beginner’s Guide to Phonetics, Jean Peccei
Mathematical model/2D

- fluid problem
- structure problem
- interface conditions
Mathematical model/2D

Fluid flow: Navier-Stokes system in ALE form

\[
\frac{D^A \mathbf{v}}{Dt} + (\mathbf{v} - \mathbf{w}_g) \cdot \nabla \mathbf{v} + \nabla p - \nu \Delta \mathbf{v} = 0 \\
\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t
\]

Structure: Linear elasticity

\[
\rho^b \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \tau_{ij}^b}{\partial x_j} = f_i, \quad \text{in } \Omega^b,
\]

- interface conditions (kinematics/dynamics)
- difficulties: contact, mesh distorsion, boundary conditions (?)
ALE derivative:

\[ \frac{DA_v}{Dt} \approx \frac{3v_{n+1} - 4\tilde{v}_n + \tilde{v}_{n-1}}{2\Delta t} \]

NS - weak reformulation \((b = (v - w_g))\)

\[ \left( \frac{3v}{2\Delta t}, z \right) + \left( [b \cdot \nabla] v, z \right) + (\nu \nabla v - pI, \nabla z) + (\nabla \cdot v, q) = \left( \frac{4\tilde{v}_n - \tilde{v}_{n-1}}{2\Delta t}, z \right) \]

Stabilization (SUPG+PSPG) \(\psi = \delta_K \left( (b \cdot \nabla)z + \nabla q \right)\):

\[ \text{LHS} + \sum\limits_K \left( \frac{3v}{2\Delta t} - \nu \Delta v + (b \cdot \nabla) v + \nabla p, \psi \right)_K = \sum\limits_K \left( \frac{4v^n - v^{n-1}}{2\Delta t}, \psi \right)_K + \text{RHS} \]

Include div-div term (LHS):

\[ \text{LHS} + \sum\limits_{K \in \mathcal{T}_h} \tau_K \left( \nabla \cdot u, \nabla \cdot v \right)_K \]
FE choice: velocity $P_2$, pressure, $P_1$
linearization of stabilized formulation
coupled with structure - strong coupling
outflow boundary (!) - modified do-nothing b.c. AVI AVI (detail)
Flow interactions with elastic structure (A. Kosik)
Use a simplified model!

Elastic structure motion - 2 dof model

\[
\begin{align*}
&M \left( \begin{array}{c}
\ddot{w}_1 \\
\ddot{w}_2 
\end{array} \right) + B \left( \begin{array}{c}
\dot{w}_1 \\
\dot{w}_2 
\end{array} \right) + K \left( \begin{array}{c}
w_1 \\
w_2 
\end{array} \right) + \left( \begin{array}{c}
F_1 \\
F_2 
\end{array} \right) + \left( \begin{array}{c}
F_{1S} \\
F_{2S} 
\end{array} \right) = 0 \\
\end{align*}
\]

\[
M = \begin{pmatrix}
m_1 + \frac{m_3}{4} & \frac{m_3}{4} \\
\frac{m_3}{4} & m_2 + \frac{m_3}{4} 
\end{pmatrix}, \quad K = \begin{pmatrix}
c_1 & 0 \\
0 & c_2 
\end{pmatrix}
\]

See Horáček, Šidlof, Švec 2002.
Coupled FSI problem

Numerical approximation of flow induced vibrations

Model M

Model F

Compare low-fidelity model, (Horacek, Svec, 2002)

<table>
<thead>
<tr>
<th></th>
<th>Model F</th>
<th>Model M</th>
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</thead>
<tbody>
<tr>
<td>shape</td>
<td>( a_f(x) )</td>
<td>( a_m(x) )</td>
</tr>
<tr>
<td>( m ) [kg]</td>
<td>( 3.274 \times 10^{-4} )</td>
<td>( 4.812 \times 10^{-4} )</td>
</tr>
<tr>
<td>( I ) [kg/m²]</td>
<td>( 1.341 \times 10^{-9} )</td>
<td>( 2.351 \times 10^{-9} )</td>
</tr>
<tr>
<td>( e ) [m]</td>
<td>( 1.133 \times 10^{-3} )</td>
<td>( 0.771 \times 10^{-3} )</td>
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</tbody>
</table>

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<th>Model M</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 ) [N/m]</td>
<td>44.8</td>
<td>56</td>
</tr>
<tr>
<td>( c_2 ) [N/m]</td>
<td>84.6</td>
<td>174.3</td>
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<tr>
<td>( f_1 ) [Hz]</td>
<td>100</td>
<td>100</td>
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<tr>
<td>( f_2 ) [Hz]</td>
<td>160</td>
<td>160</td>
</tr>
</tbody>
</table>
Numerical results (Model F)

\[ U = 0.2\text{m/s} \]

\[ U = 0.3\text{m/s} \]

\[ U = 0.34\text{m/s}, \ Q \approx 0.1\text{l/s} \]
Numerical results

Numerical approximation of flow induced vibrations

Compare low-fidelity model, (Horacek, Svec, 2002)
Inlet: velocity profile
Numerical results (Model M)

\[ U = 0.575 \text{m/s} \]

\[ U = 0.6 \text{m/s} \]

\[ U = 0.625 \text{m/s}, \quad Q \approx 0.15 \text{l/s} \]
Numerical results

Quantitative comparison: flutter velocity

![Graph showing quantitative comparison of flutter velocity for different values of $g_0$. The graph includes data for female and male subjects, both from experimental and FEM simulations.](image)
Numerical results

Quantitative comparison: pressure
Boundary condition

- **Outlet boundary condition** - modified do-nothing bc
- **Inlet boundary condition**
  - either: flow velocity \((U_0, 0)\)
  - or: pressure \(\Delta p\)
Coupled FSI problem

Numerical approximation of flow induced vibrations

Compare low-fidelity model, (Horacek, Svec, 2002)
Inlet: pressure
Numerical results (Model F, pressure drop)

- Increase of $\Delta p \rightarrow$ ael instability (AVI)
- but instability for non-physical values ($\Delta p = 10kPa, U_{max} \approx 180m/s$)
- low fidelity model: ael instability for $0.4m/s$
Pressure drop problem

Natural boundary condition - assumed to be more realistic, but it is not ...

- prescribed pressure (velocity varies)
- time dependent - very high variations of inlet velocity
- outlook: use a combination of inlet velocity/inlet pressure b. c.
Conclusion

- Aeroelastic simulations of flow interacting within the vibrating vocal fold was performed.
- The qualitative and quantitative results were discussed.
- The choice of appropriate boundary conditions were discussed.