Summer School and worshop« Wave in flows »
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Gravity wave propagation in inhomogeneous media : wave scattering and interference process

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Vincent REY, hydrodynamics
RESEARCH ACTIVITIES

- Wave propagation and transformations in the coastal zone

- Sediment transport and morphodynamics

Applications

Environmental nearshore dynamics

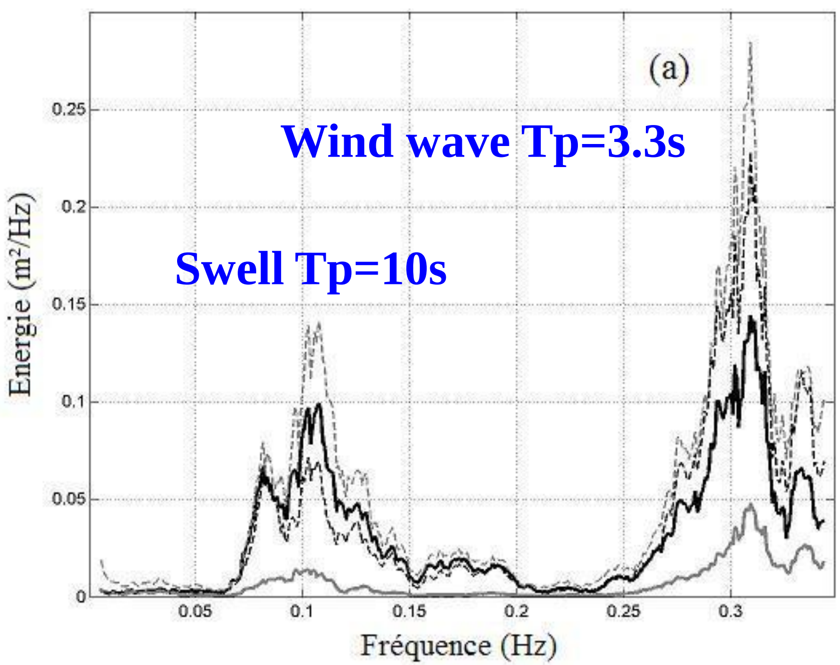
Ocean and coastal engineering

CONTENTS

Water waves : sea states and wave models

Wind wave (+ swell)

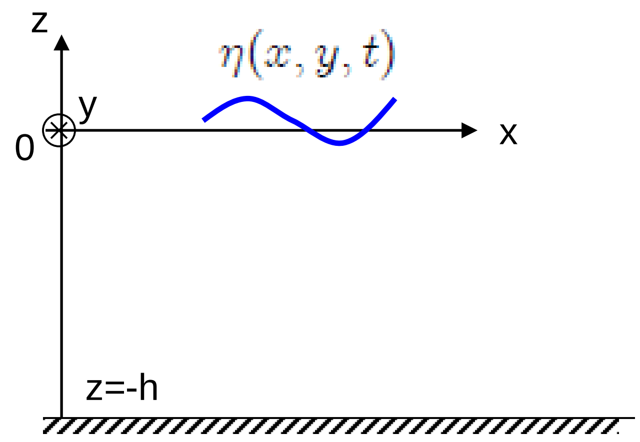
Swell (quasi-sinusoidal shape)



Sea states:
Waves characteristics : Hs, Tpeak, etc..

Wave models
- Monochromatic waves (phase-resolving models)
- Spectral waves (phase averaged models, (based on the energy flux conservation)

MONOCHROMATIC WAVES : STOKES THEORIES



Hypothesis

- Inviscid fluid : Euler equations

- Irrotational motion

→ Potential flow $\Phi(x,y,z,t)$ and $\vec{v} = \vec{\nabla}\Phi$

Conditions for the potential $\Phi(x,y,z,t)$

- Laplace's equation (η>z>-h): $\nabla^2\Phi = 0$

- Free surface conditions (z=η) :

  - Kinematic : $$\frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} - \frac{\partial \Phi}{\partial z} = 0$$

  - Dynamic (Bernoulli) : $$g\eta + \frac{1}{2}\vec{v}^2 + \frac{\partial \Phi}{\partial t} = 0$$

- Bottom impermeability (z=–h) : $$\frac{\partial \Phi}{\partial z} = 0$$

using $\vec{v}.\vec{\nabla}\left(\frac{\partial\Phi}{\partial t}\right)=\frac{1}{2}\frac{\partial\vec{v}^2}{\partial t}$

Total differentiation of the dynamic free surface equation and choice p=0 for the surface pressure lead to the free surface condition (z=η) :

$$g\frac{\partial\Phi}{\partial z}+\frac{\partial\vec{v}^2}{\partial t}+\frac{\partial^2\Phi}{\partial t^2}+\frac{1}{2}\vec{v}.\vec{\nabla}(\vec{v}^2)=0$$

REMARKS

- The free surface condition is NON-LINEAR

- Expression is given at z=η, which depends on both time and space

Approximations:
- Linear : 1st order Stokes wave or Airy wave
- Perturbation methods : Stokes waves at higher orders

NON-LINEAR solutions : Asymptotic expansion of η and Φ

$$\begin{cases} \eta = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \varepsilon^3\eta_3 + O(\varepsilon^4) \\ \Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 + O(\varepsilon^4) \end{cases}$$

Free surface condition for Φ and Bernoulli expansion for η expressed at z=0 using taylor expansion:

$$f(x,\eta,t) = f(x,0,t) + \eta\left[\frac{\partial f}{\partial z}\right]_{z=0} + \frac{\eta^2}{2}\left[\frac{\partial^2 f}{\partial z^2}\right]_{z=0} + \dots$$

Free surface (combined kinematic and dynamic (Bernoulli) conditions:

O(ε): $\frac{\partial^2\Phi_1}{\partial t^2} + g\frac{\partial\Phi_1}{\partial z} = 0$

O(ε²): $\frac{\partial^2\Phi_2}{\partial t^2} + g\frac{\partial\Phi_2}{\partial z} = -\eta_1\left(\frac{\partial^3\Phi_1}{\partial t^2\partial z} + g\frac{\partial^2\Phi_1}{\partial z^2}\right) - \frac{\partial}{\partial t}\left[\left(\frac{\partial\Phi_1}{\partial x}\right)^2 + \left(\frac{\partial\Phi_1}{\partial z}\right)^2\right]$

Bernoulli condition :

O(ε): $-g\eta_1 = \frac{\partial\Phi_1}{\partial t}$

O(ε²): $-g\eta_2 = \frac{\partial\Phi_2}{\partial t} + \eta_1\frac{\partial^2\Phi_1}{\partial z\partial t} + \frac{1}{2}\left[\left(\frac{\partial\Phi_1}{\partial x}\right)^2 + \left(\frac{\partial\Phi_1}{\partial z}\right)^2\right]$

1st Order STOKES Wave : Airy wave

Wave steepness ka<<1:

Free surface boundary condition (z=0) : $g\frac{\partial \Phi}{\partial z} + \frac{\partial^2 \Phi}{\partial t^2} = 0$

For a free surface deformation of the form $\eta(x,t) = a\sin(\omega t - kx)$

The potential $\Phi(x,z,t)$ is given by

$$\Phi(x,z,t) = \frac{a\omega}{k}\frac{\cosh[k(z+h)]}{\sinh(kh)}\cos(\omega t - kx)$$

Elliptic trajectories

$$X = a\frac{\cosh[k(z+h)]}{\sinh(kh)}\cos(\omega t - kx) = X_0\cos(\omega t - kx)$$

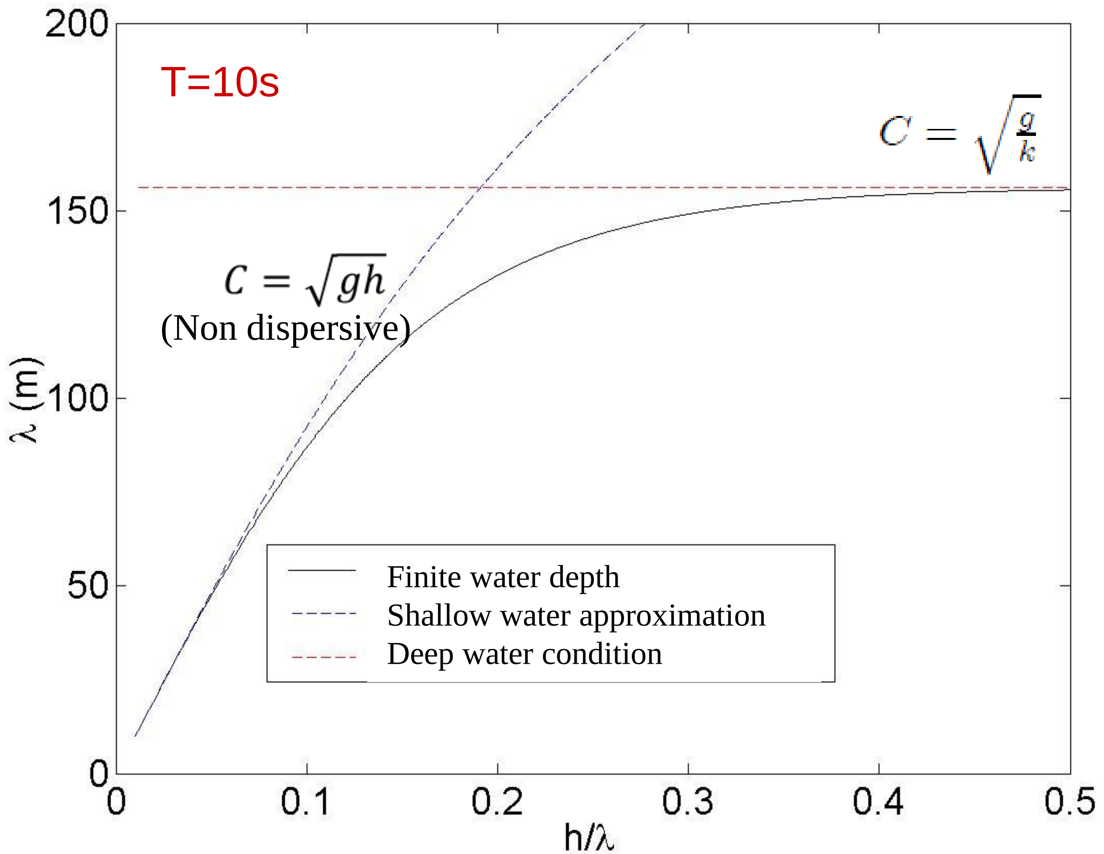
$$Z = a\frac{\sinh[k(z+h)]}{\sinh(kh)}\sin(\omega t - kx) = Z_0\sin(\omega t - kx)$$

$$\left[\frac{X}{X_0}\right]^2 + \left[\frac{Z}{Z_0}\right]^2 = 1$$

The wavenumber k verifies the dispersion relation $\omega^2 = gk\tanh(kh)$

The wave celerity (phase celerity) is given by $C = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kh)}$

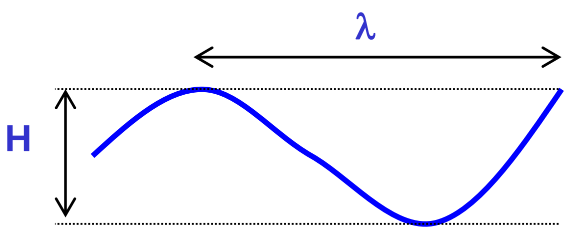
The wave propagation is **dispersive**



shallow water

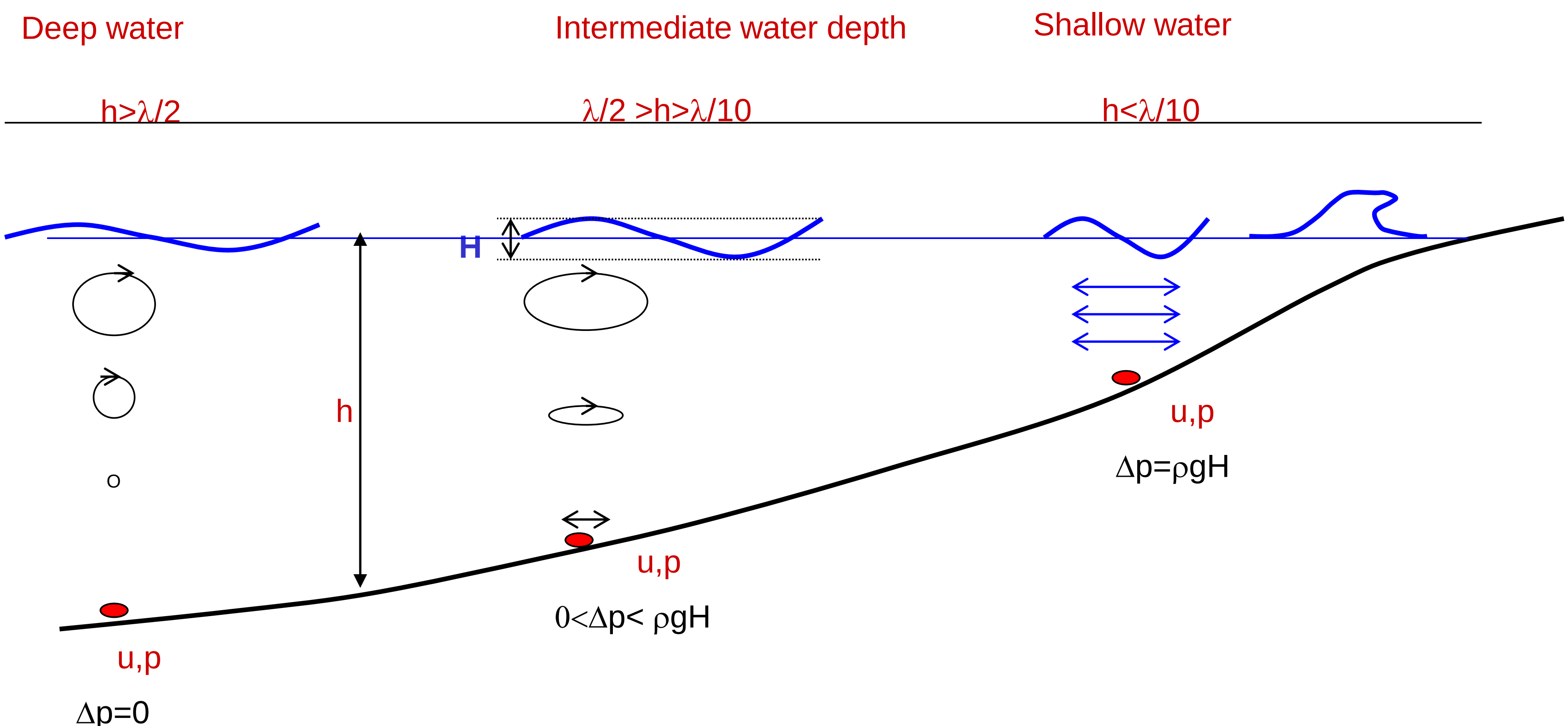
deep water

CHARACTERISTIC DATA and CLASSIFICATIONS



$T \approx 3-6s$ **wind waves**

$T \approx 8-14s$ **Swell**



In deep water conditions, no wave impact near the bottom

In shallow water conditions, hydrostatic conditions (in the linear approximation)

Wave energy :

Potential energy by unit volume: $d^3E_p = \rho g z dx dy dz$

Potential energy by unit wavelength λ and width dy for a monochromatic wave Propagating along the x-axis :

$$E_p = \int_0^\lambda \rho g \frac{\eta^2(x)}{2} dx$$

Airy wave:

$$E_p = \frac{\rho g a^2}{4}\lambda$$

Kinetic energy by unit wavelength λ and width dy :

$$E_c = \frac{1}{2}\int_{x=0}^{\lambda}\int_{z=-h}^{\eta} \rho \vec{v}^2 dx dz$$

$$E_c = \frac{\rho g a^2}{4}\lambda$$

Total energy by unit length dx and width dy :

$$E = \frac{\rho g a^2}{2}$$

Mean energy flux across the y0z plane, normal to the direction of propagation:

$$E_t = \frac{1}{T}\int_t^{t+T}\int_{x=-h}^{\eta} p\vec{v}.\vec{n} dz dt \quad \text{with} \quad p = -\rho\frac{\partial \Phi}{\partial t} - \frac{1}{2}v^2$$

Airy wave :

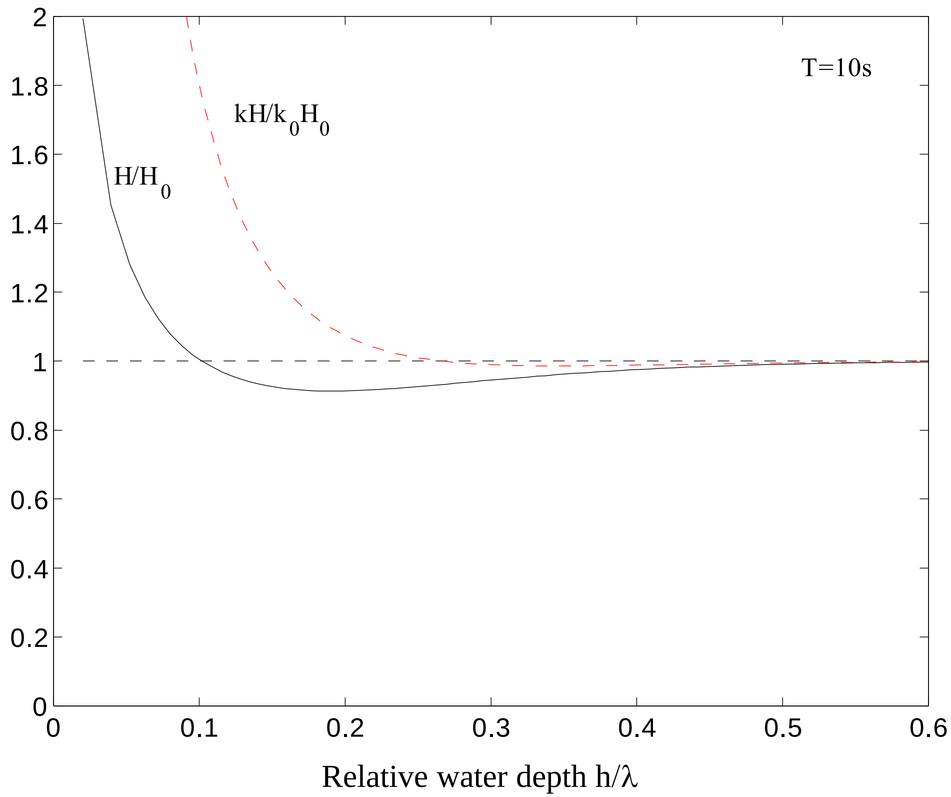
$$E_t = \frac{\rho g a^2}{4} C \left[1 + \frac{2kh}{\sinh 2kh}\right] = \frac{\rho g a^2}{2} C_g = E C_g \quad \text{with} \quad C_g = \frac{\partial \omega}{\partial k} = \frac{1}{2} C \left[1 + \frac{2kh}{\sinh 2kh}\right]$$

Wave energy is transported at the group velocity

Wave propagation on a gentle slope, the « shoaling »

Energy flux conservation for a progressive wave:

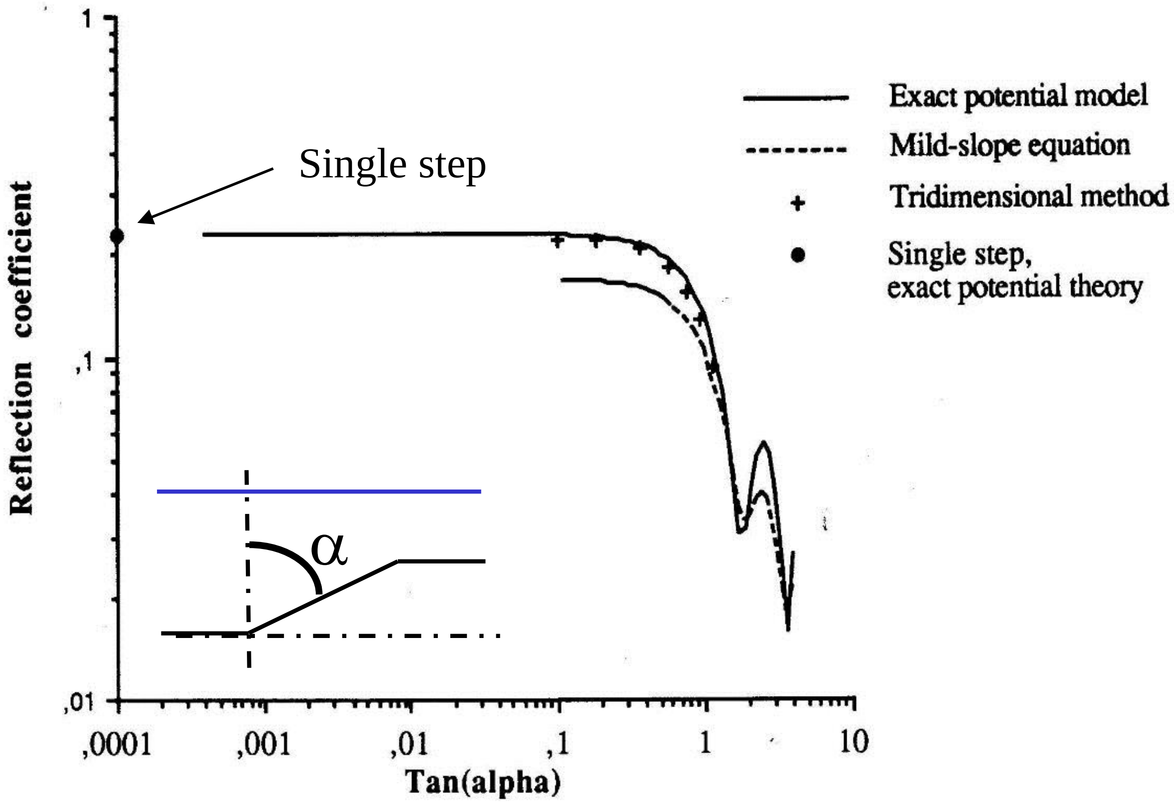
$$\frac{\partial E_t}{\partial x} = 0 \quad \Longrightarrow \quad \left(\frac{a}{a_0}\right)^2 = \left(\frac{H}{H_0}\right)^2 = \frac{C_{g0}}{C_g} = \frac{1}{\tanh kh + \frac{kh}{\cosh^2 kh}}$$



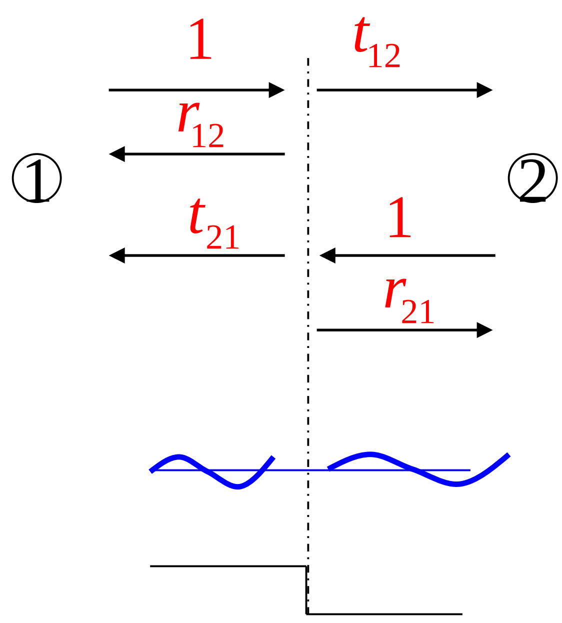
Wave steepness kH rapidly increases when h decreases

→ Non linear effects

Effect of a bed slope on the wave reflection



Index discontinuity:
Partial reflection



Gentle slopes ➡ weak reflection (< 1% in terms of energy)

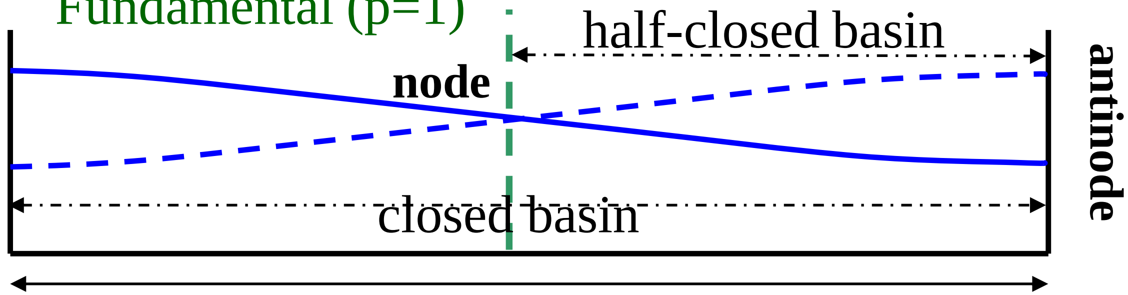
Bottom step ➡ reflection cannot be neglected

Rey, 1992

Resonant interactions (2D cases)

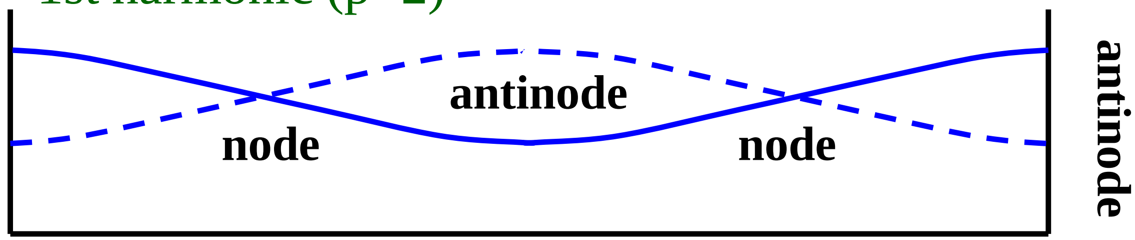
Standing waves : basin oscillations or « seiching »

Fundamental (p=1)



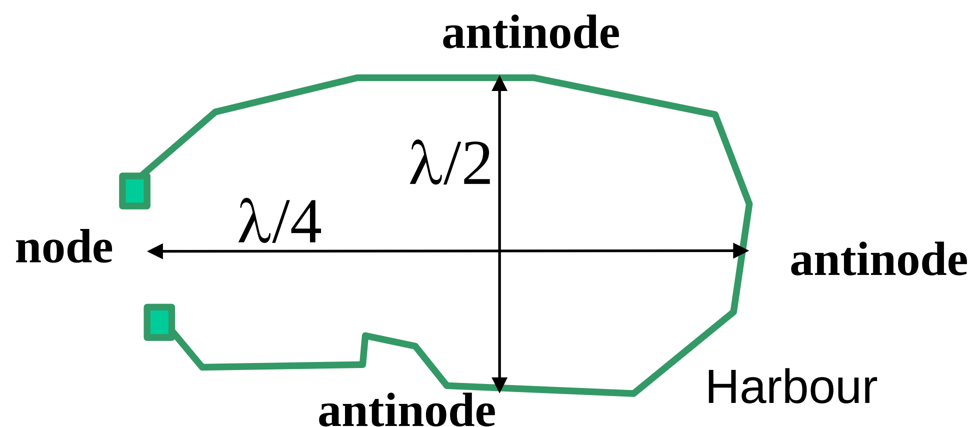
$$L = \frac{p\lambda}{2}$$

1st harmonic (p=2)



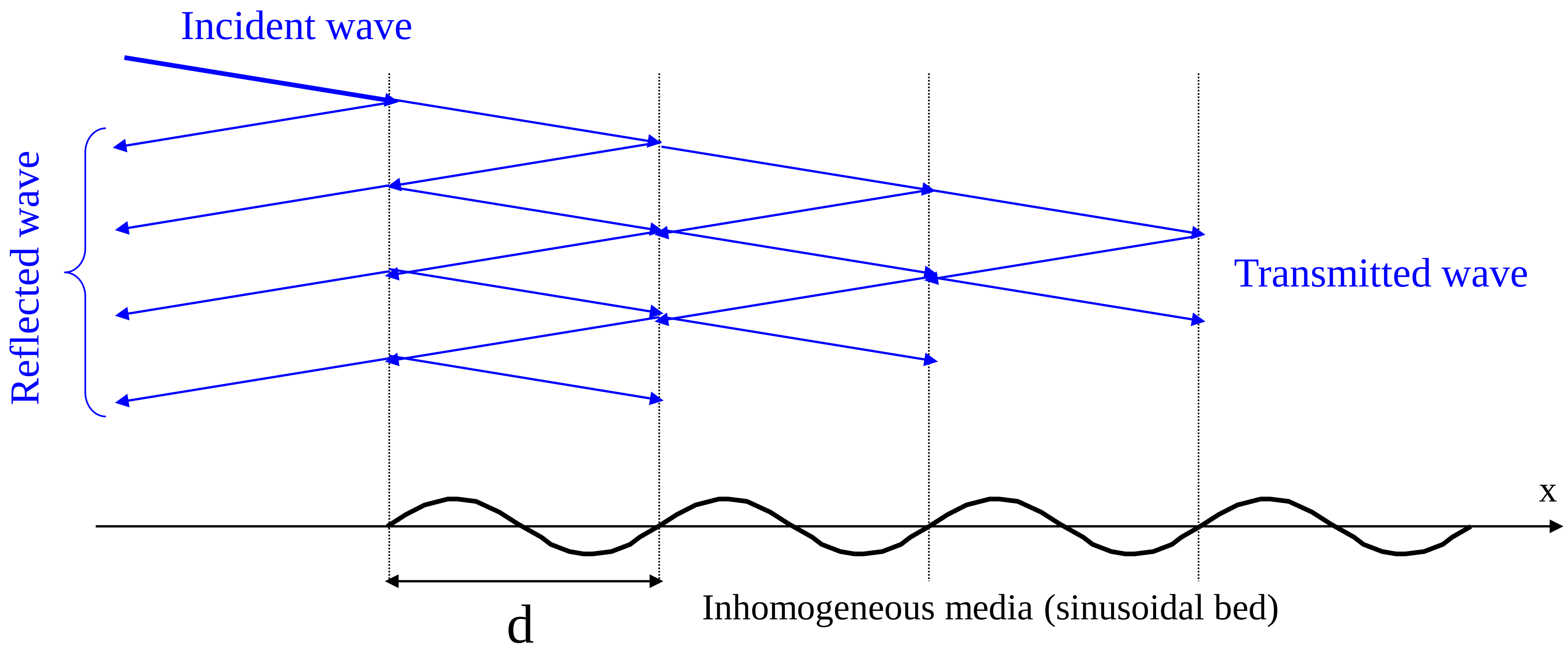
Harbour oscillations

Periods of seiching :
Several to tens minutes



Partially standing waves : Interference processes

Interferences : physical approach



Phase lag after one return path: $\Delta\varphi = 2\frac{2\pi}{\lambda}\,\mathrm{d}$

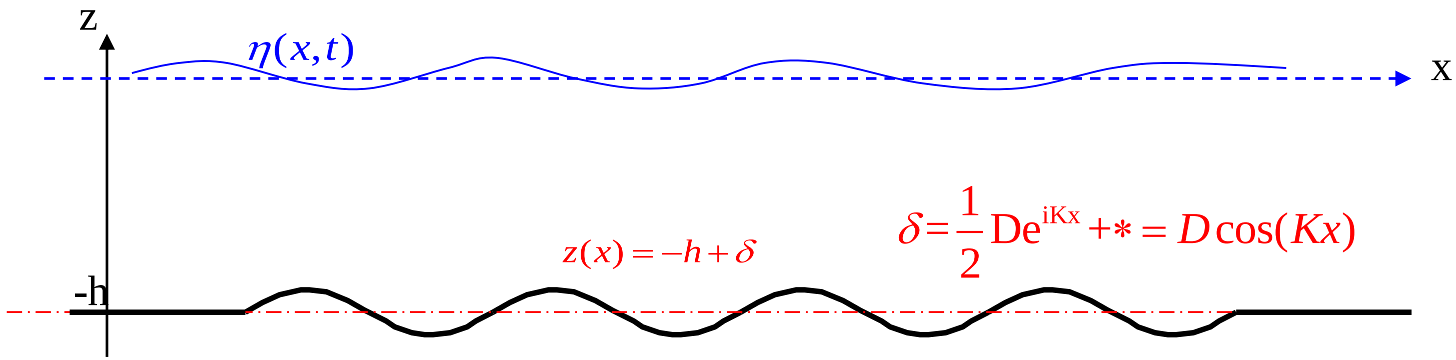
Phase matching → Reflection « allowed »

Opposition of phases → Reflection «forbidden »

➡ **Interference process**

➡ **Maximum of reflection for λ=2d (Bragg Resonance)**

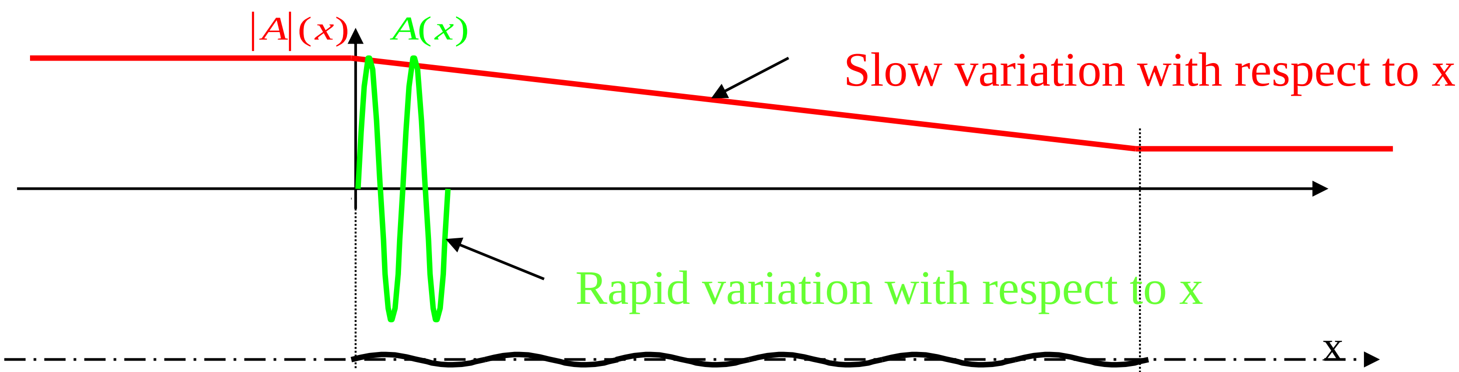
Reflection forced by the bottom boundary condition



Bottom boundary condition $\frac{\partial \Phi}{\partial z} = -\frac{\partial \delta}{\partial x}\frac{\partial \Phi}{\partial x}$ for $z = -h + \varepsilon\delta$

Taylor expansion (z=-h): $\frac{\partial \Phi}{\partial z} = \frac{\partial}{\partial x}\left[\delta \frac{\partial \Phi}{\partial x}\right] + \ldots$

If K=2k : $\frac{\partial}{\partial x}\left[\delta \frac{\partial \Phi}{\partial x}\right] \propto \cos(\omega t + kx)$ ➡ **reflected wave**



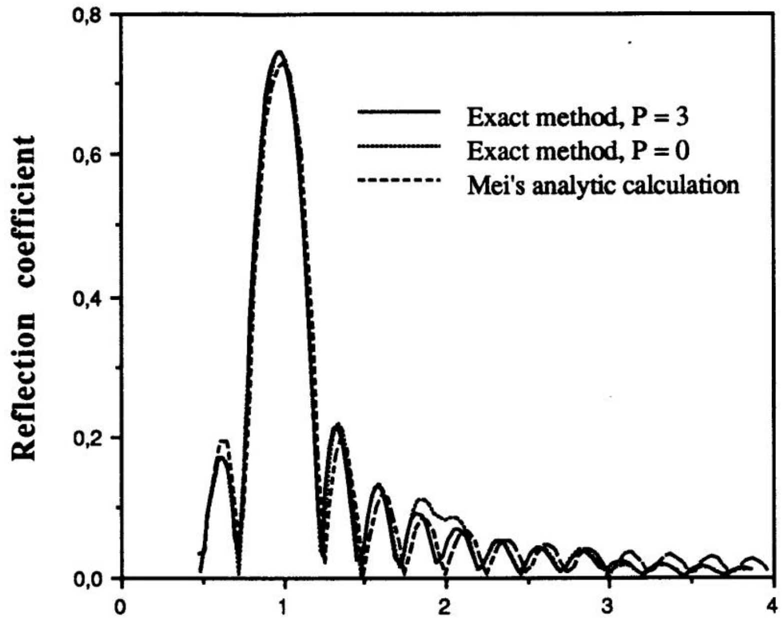
➡ **Multi-scale expansion method:** slow variables $x_1 = \varepsilon x$ and $t_1 = \varepsilon t$

$$\Phi(x,z,t) = \varepsilon\Phi_1(x,z,t,x_1,t_1) + \varepsilon^2\Phi_2(x,z,t,x_1,t_1) + \ldots$$

Mei, 1983

Reflection forced by the bottom boundary condition

Sinusoidal bed

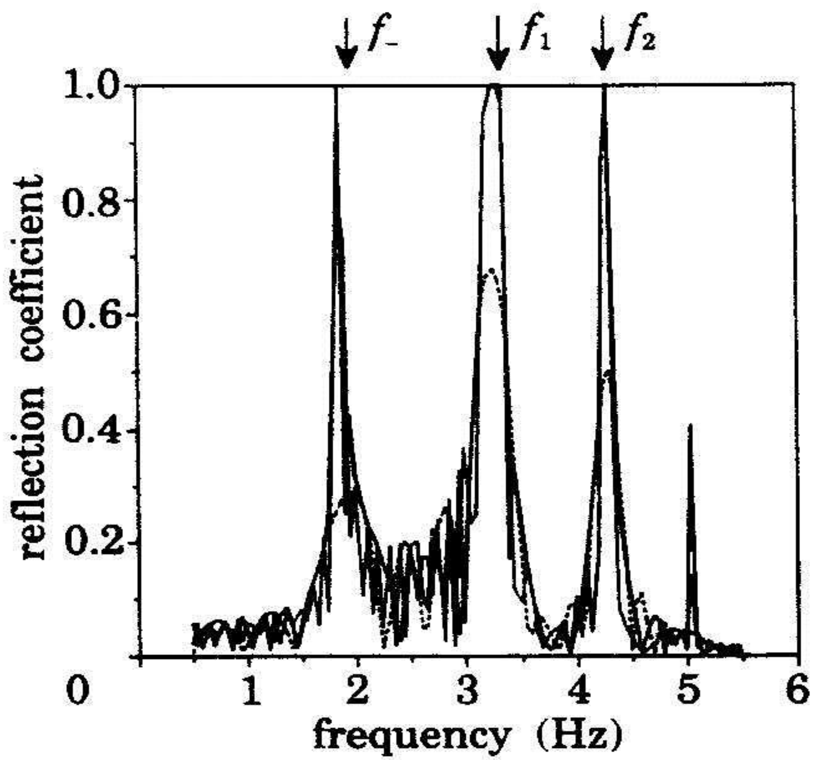


2k/K

Bragg resonance for K=2k :

➡ **Strong reflection at the Bragg conditions**
Oscillations on both parts due to the finite length of the bed

Doubly sinusoidal bed



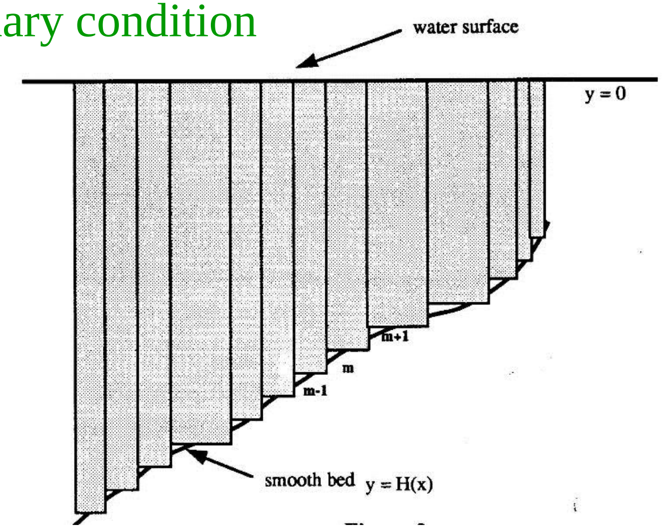
1st order and subharmonic (2nd order) Bragg resonance

➡ **High interaction at low frequency even if « 2nd order » Bragg**

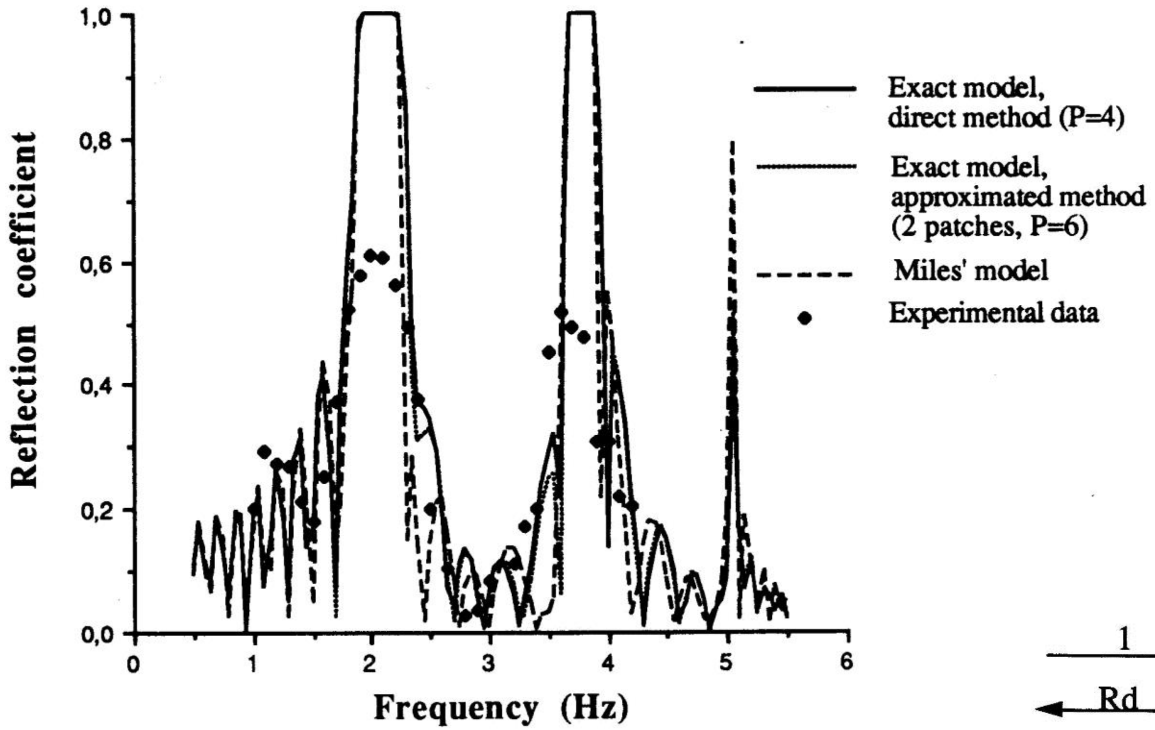
➡ **Reflection tends to 1 for increasing bed length**

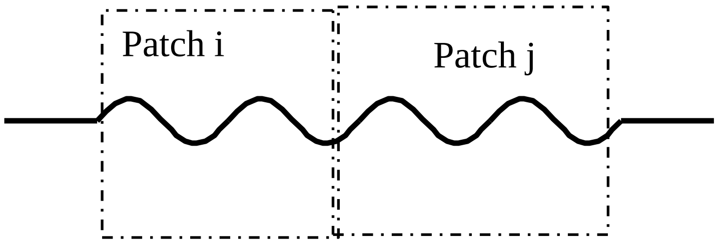
Belzons et al, 1991; Rey, 1992

Reflection forced by the bottom boundary condition

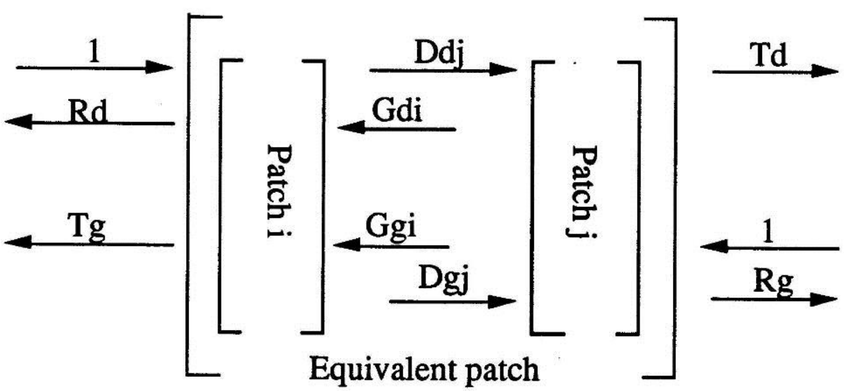


Doubly sinusoidal bed
Interference process :
Analogy with the PF interferometer



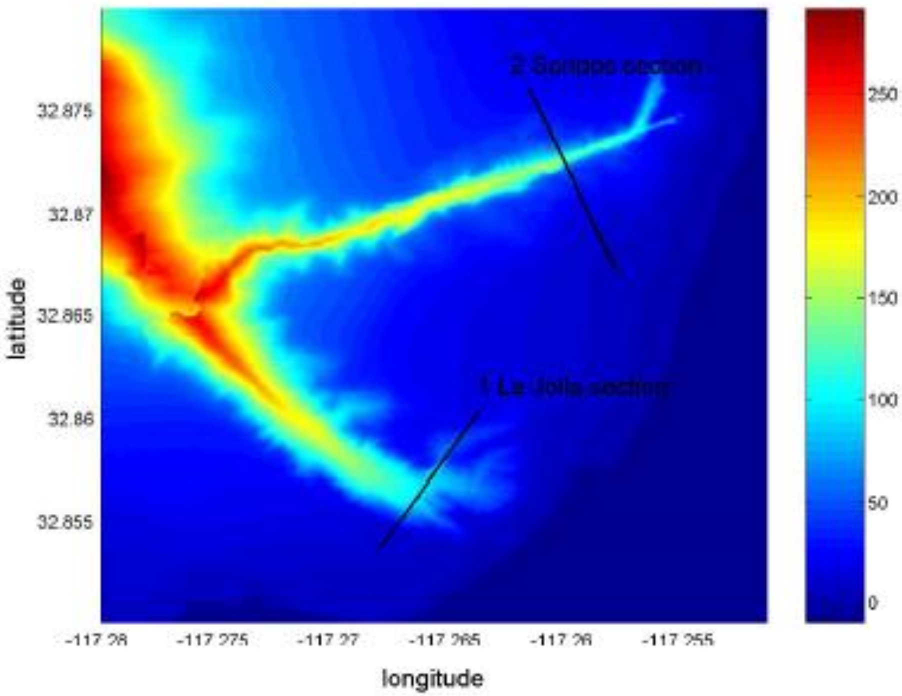


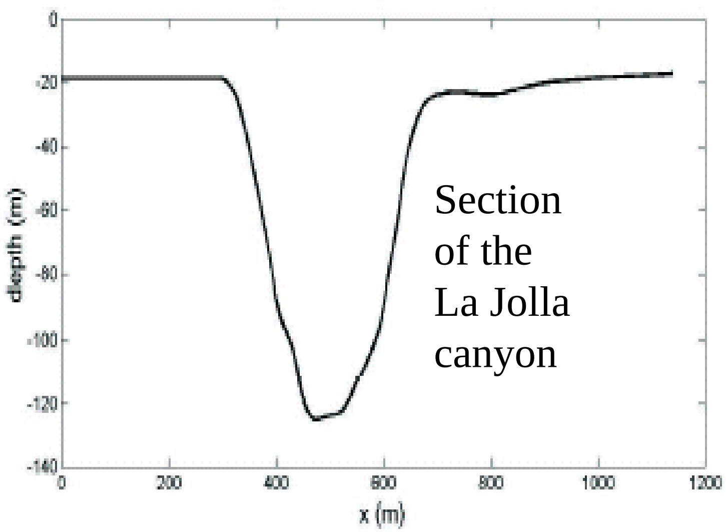
interference process



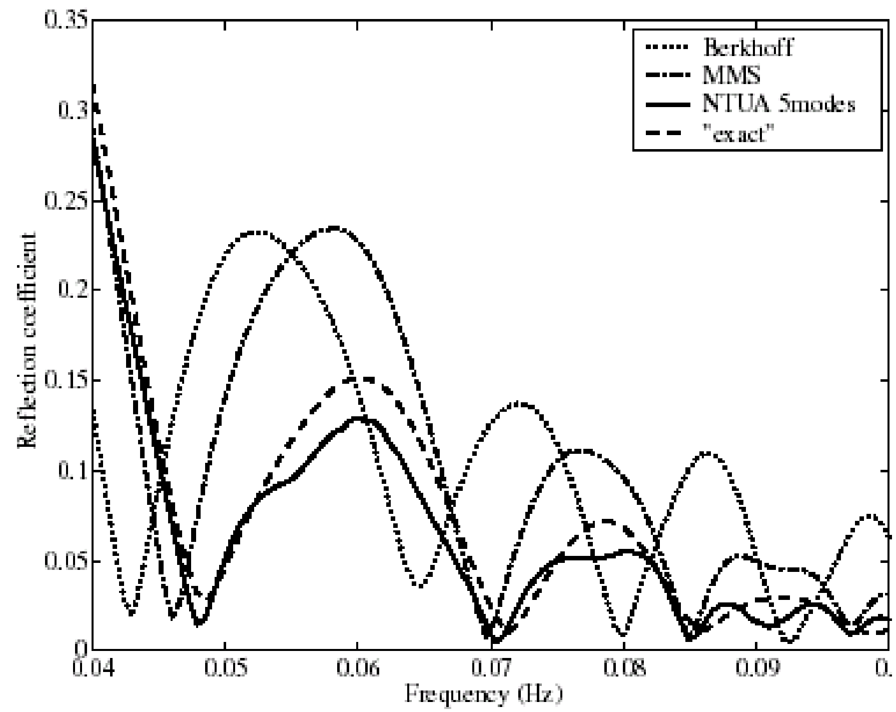
Guazzelli et al, 1992; Rey, 1992

Reflection in the presence of canyon : the La Jolla canyon (califormia, USA)





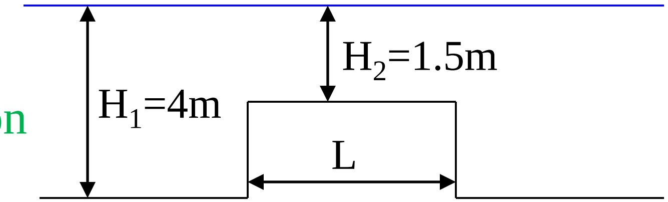
Reflection coefficient for a normally incident wave



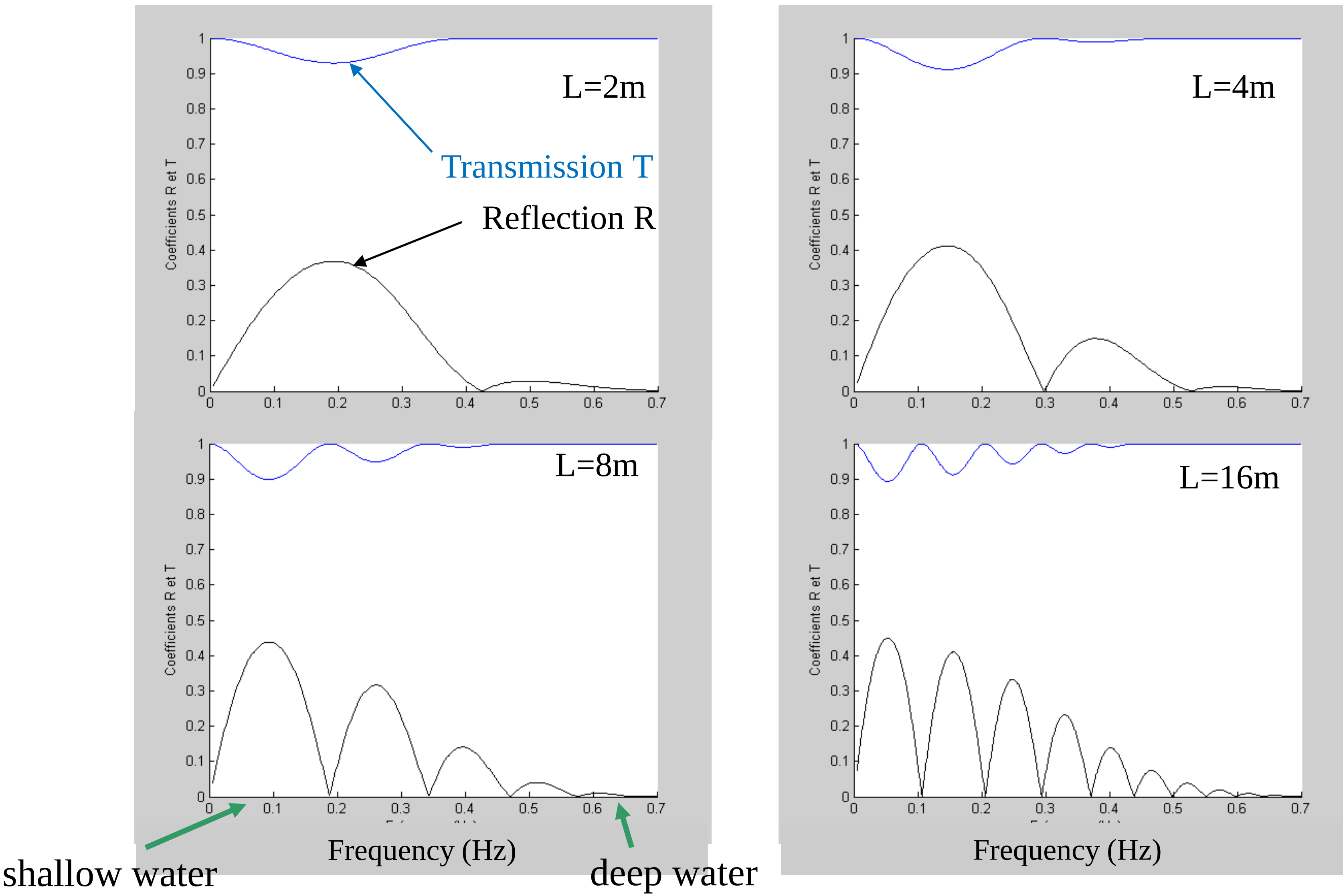
Magne et al, 2007

Application to nearshore structures:

Rectangular bar : Influence of the length on the reflection



$$1 = R^2 + T^2$$



Application to nearshore structures:

Expression of the velocity potential : presence of evanescent modes

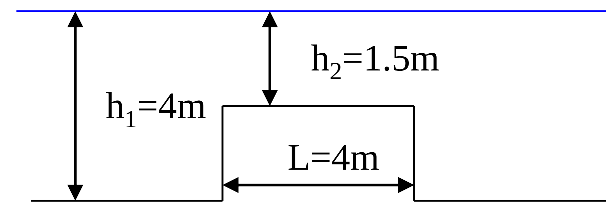
$$\Phi(x,y,z,t) = \phi(x,z)\ e^{i\omega t}$$

$$= \left[\left[A^- e^{-ik_x x} + A^+ e^{+k_x x}\right] \chi(z) + \underbrace{\sum_{n=1}^{\infty} \left[B_n^- e^{-k_{xn} x} + B_n^+ e^{+k_{xn} x}\right] \psi_n(z)}_{evanescent} \right] e^{i\omega t}$$

with

$$\chi(z) = \cosh k(h+z) = \psi_0(z) = \cos k_0(h+z)$$

$$\psi_n(z) = \cos k_n(h+z)$$

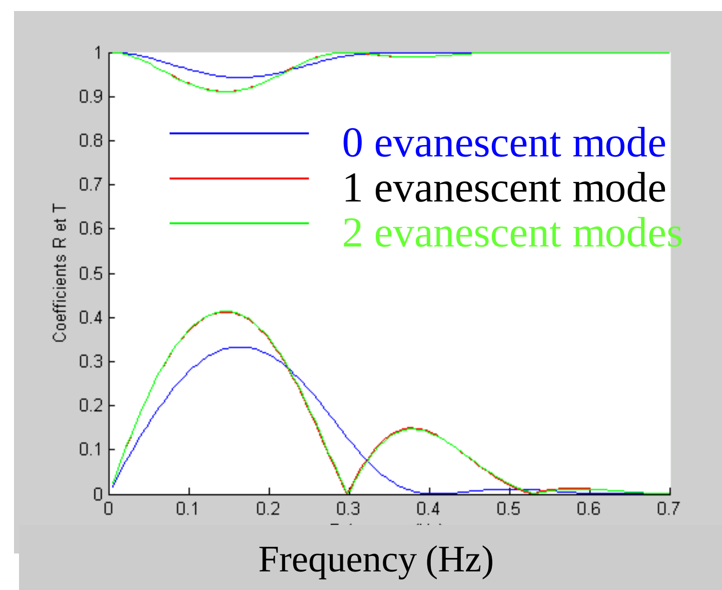


Integral matching method, at a step:

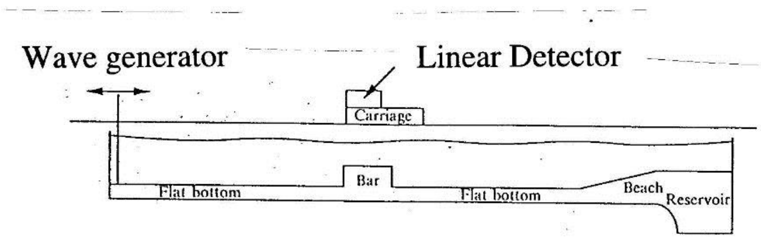
Pressure: $$\int_{-h_2}^{0} \phi_1.\psi_{2,n} dz = \int_{-h_2}^{0} \phi_2.\psi_{2,n} dz$$

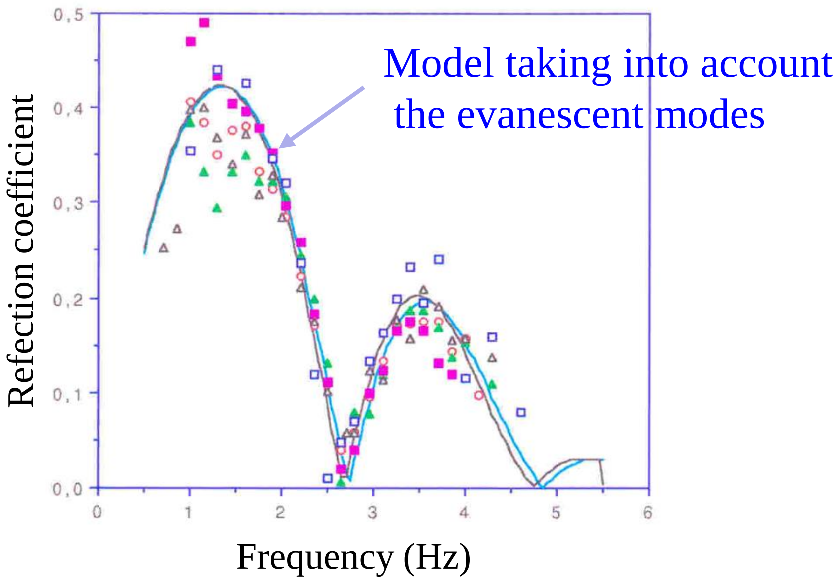
Velocity: $$\int_{-h_1}^{0} \frac{\partial \phi_1}{\partial x}.\psi_{1,n} dz = \int_{-h_2}^{0} \frac{\partial \phi_2}{\partial x}.\psi_{1,n} dz$$

for $n = 0, .., P$

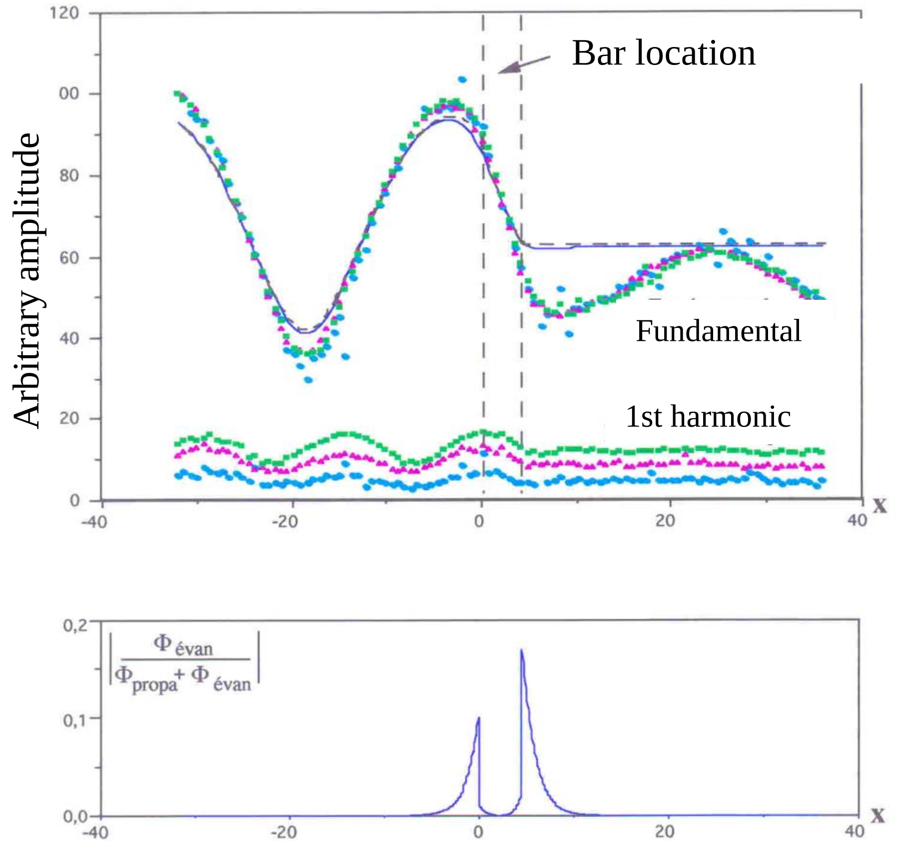


Weight of evanescent modes





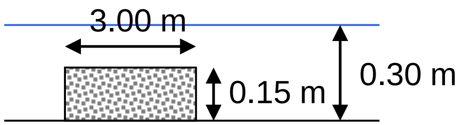
f = 1 Hz



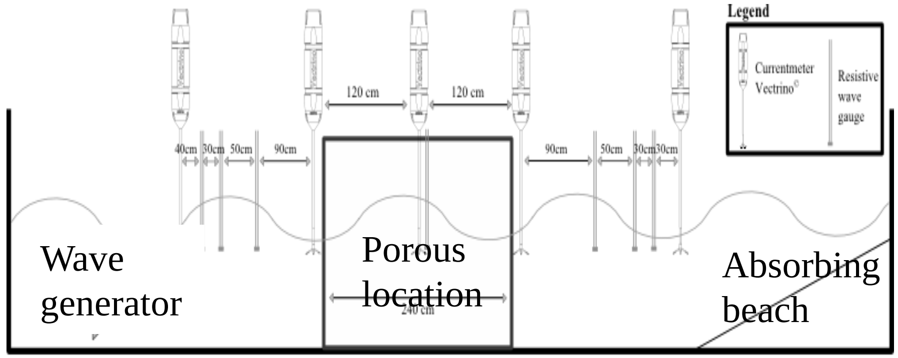
➡ **Significant weight at the vicinity of the structure**
Evanescent modes involved in the interference process

Rey, et al 1992

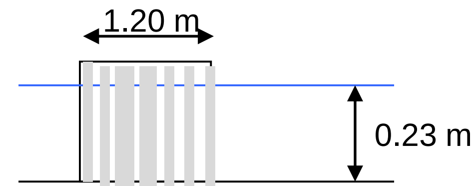
Application to nearshore structures: Porous structures: reflection and **dissipation**



Porosity γ=0.3



Experimental set-up, SeaTech wave tank, UTLN



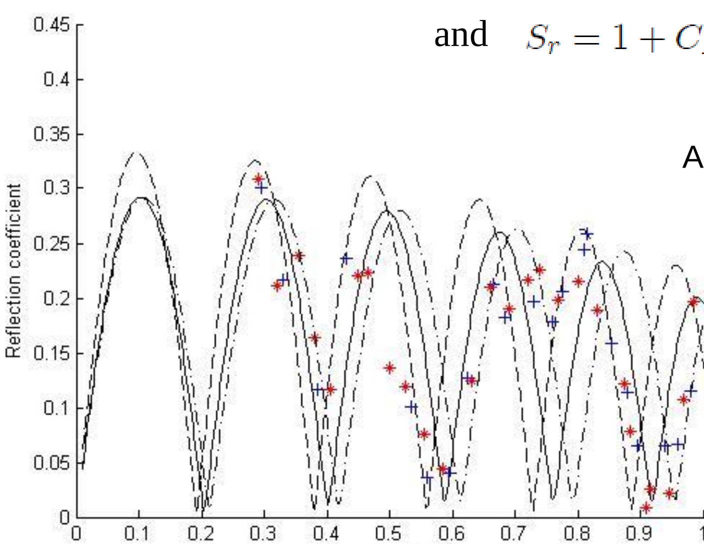
D=0.050 m
Porosity γ=0.7

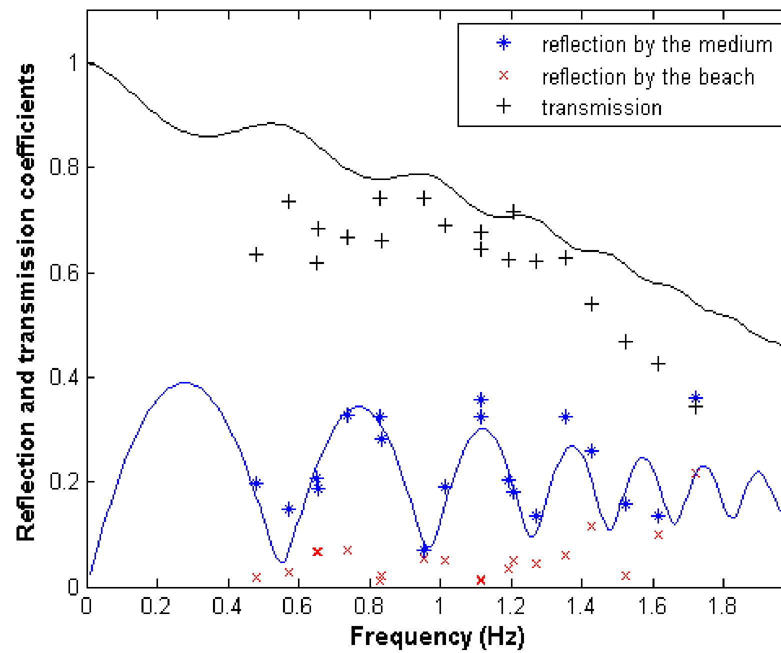
Dispersion relation $Z\omega^2 = igk\tanh(kh)$

with $Z = f_R + iS$

and $S_r = 1 + C_M \frac{(1-\gamma)}{\gamma}$

Added mass



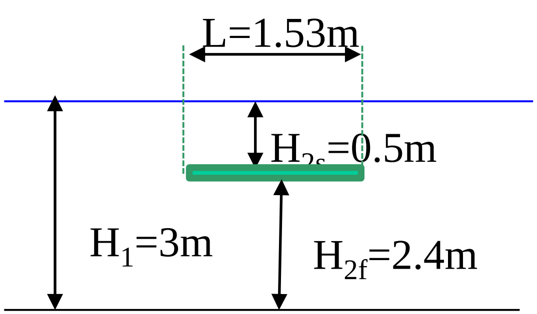


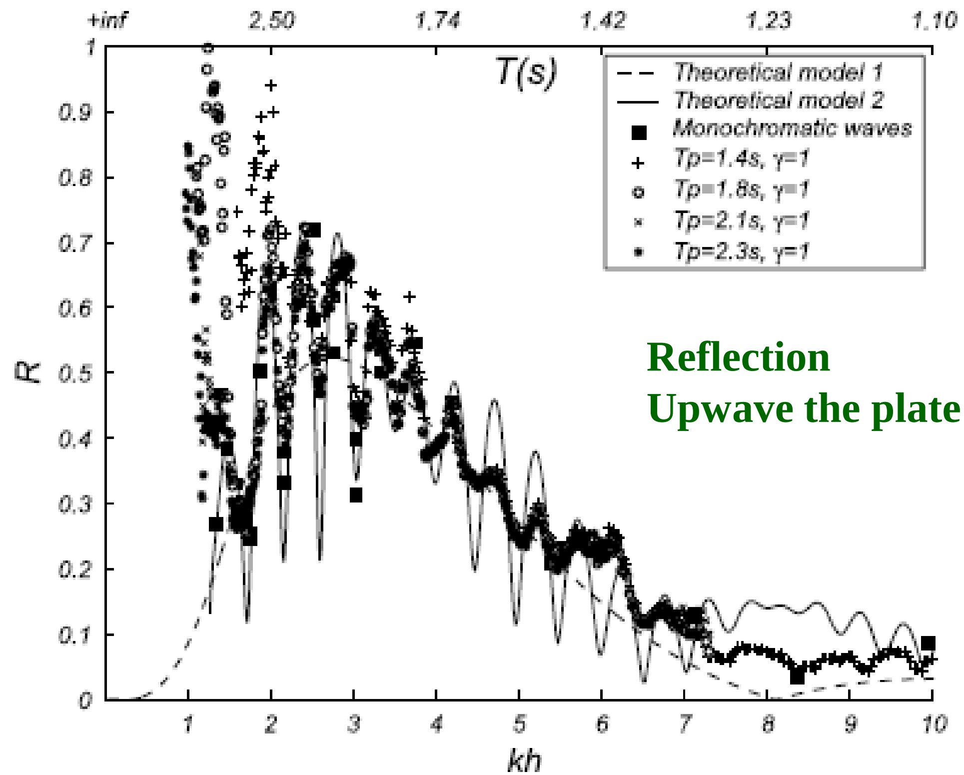
David de Drézigué et al, 2013

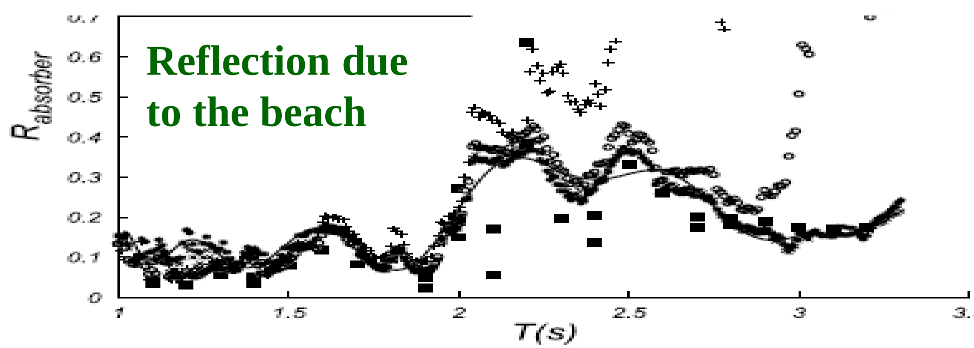
Arnaud et al, 2017

Application to nearshore structures:

Submerged plate : Interference process and Influence of the beach on the reflection





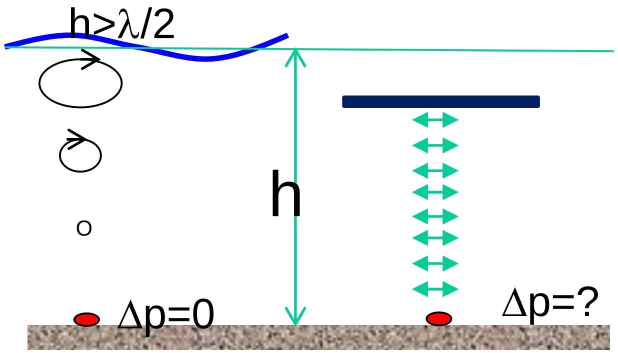


Interference process above the plate
Oscillating behaviour due to reflection from the beach (absorber)

Rey and Touboul, 2011

Application to nearshore structures:

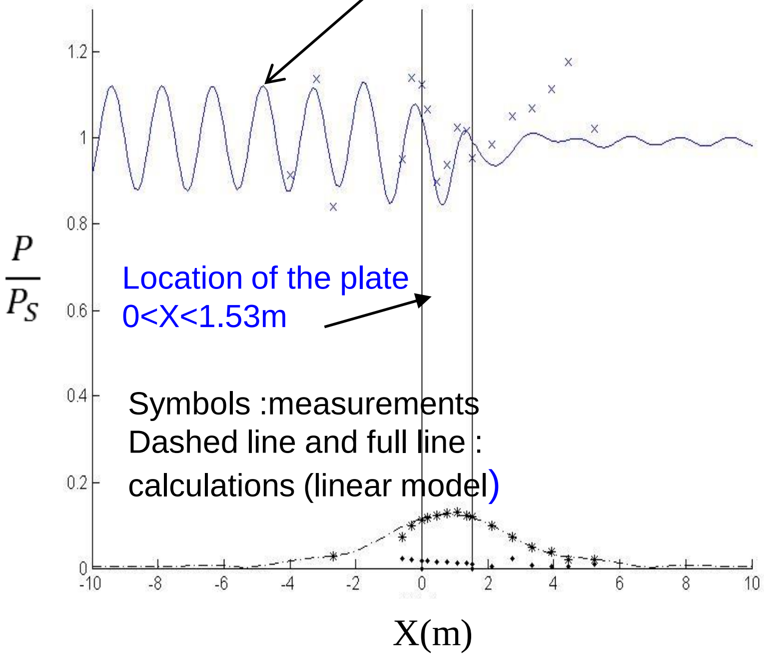
Submerged plate : bottom induced pressure



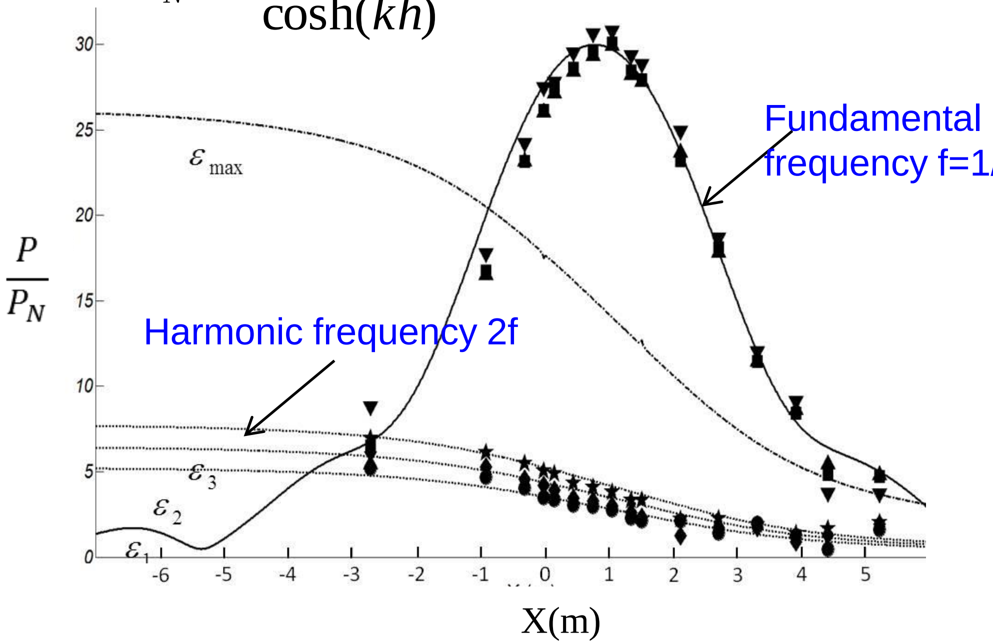
Free surface deformation and bottom pressure amplitudes,
T=1.4s, a=54mm

Partially standing wave

$$P_S = \rho g a_i$$



$$P_N = \frac{\rho g a_i}{\cosh(kh)}$$



- 1st order bottom pressure amplitude (frequency f) 30 times the bottom pressure due to the incoming wave
- « Longuet-Higgins effect » at 2nd order (frequency 2f)

Touboul and Rey, 2012

Application to wave energy device:

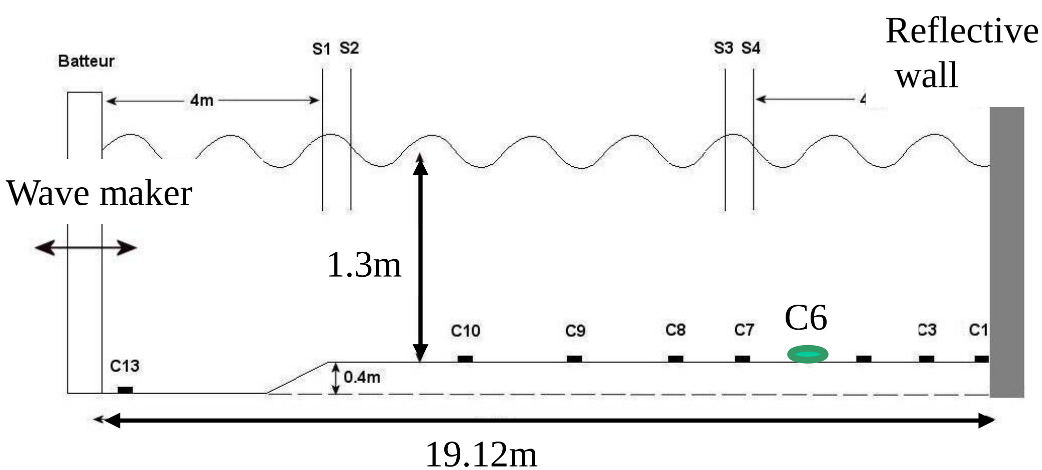
bottom induced pressure at twice the wave frequency for « deep water » conditions

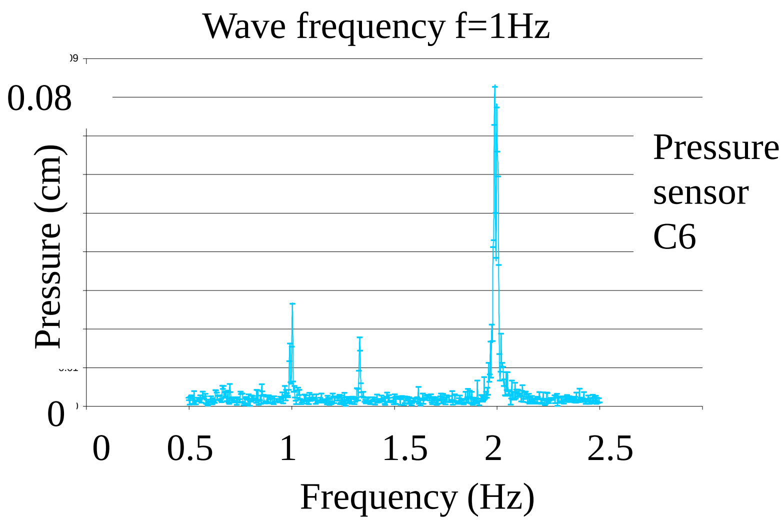
Non linear free surface condition at z=η: $g\frac{\partial \Phi}{\partial z} + \frac{\partial \vec{v}^2}{\partial t} + \frac{\partial^2 \Phi}{\partial t^2} + \frac{1}{2}\vec{v}.\vec{\nabla}(\vec{v}^2) = 0$

Perturbation method (for 2nd order Stokes waves): $\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + O(\epsilon^2)$

$$\begin{cases} \Phi_1 = \frac{a^-\omega}{k}e^{kz}\cos[\omega t - kx] + \frac{a^+\omega}{k}e^{kz}\cos[\omega t + kx] \\ \Phi_2 = -a^-a^+\sin[2\omega t] \end{cases}$$

$p = -\rho\frac{\partial \Phi}{\partial t} - \frac{1}{2}v^2$ ➡ $p = \rho\omega a^-a^+\cos(2\omega t)$ for z=-h (in fact for any z)



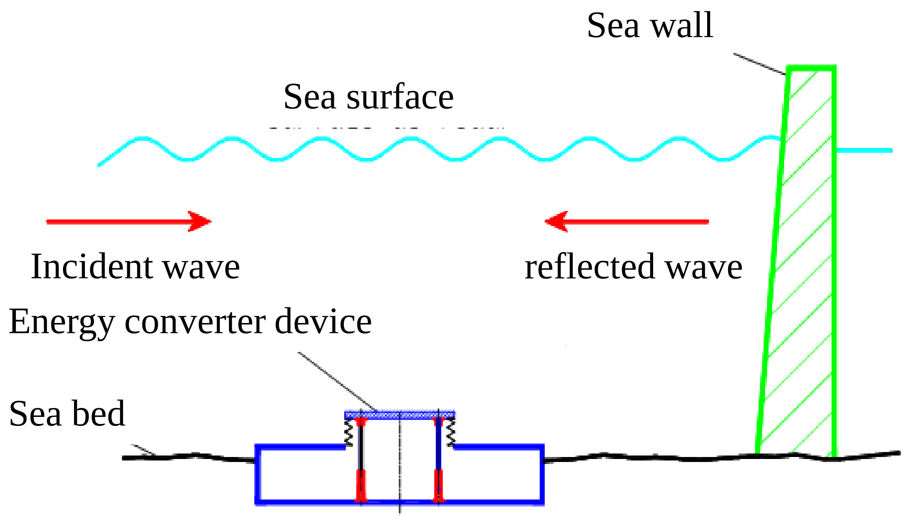


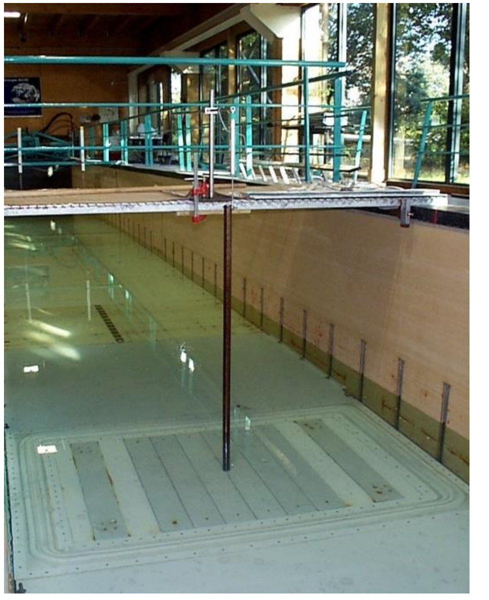
Jarry, 2009

➡ high peak pressure at twice the wave frequency (Longuet-Higgins, 1950))

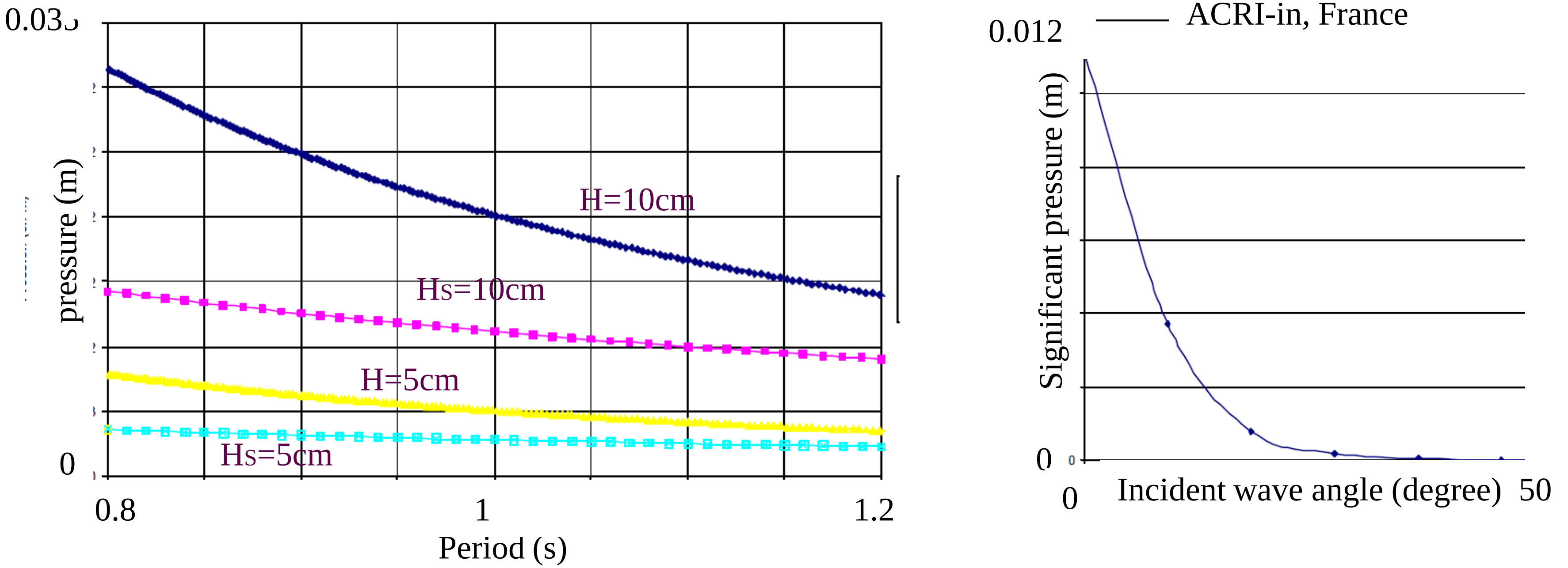
Application to wave energy device:

Real sea states





Experimental set-up, ACRI-in, France



➡ « Longuet-Higgins » effect significantly decreases for wave frequency and direction spreading

Jarry, 2009

Refraction – diffraction

Slow changes of wave properties :

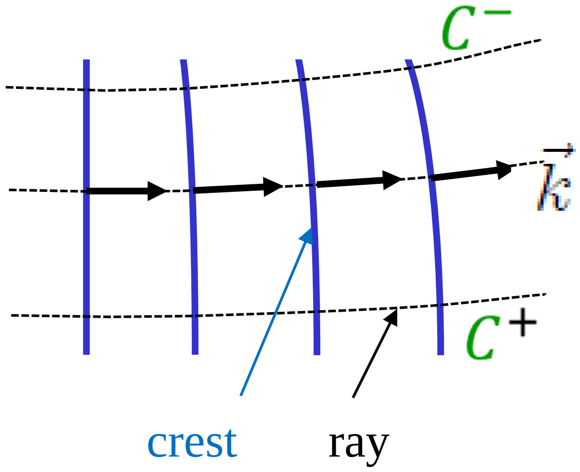
Plane waves, ray theory

Rapid change of amplitude and/or direction :

No more plane waves, diffraction parameter

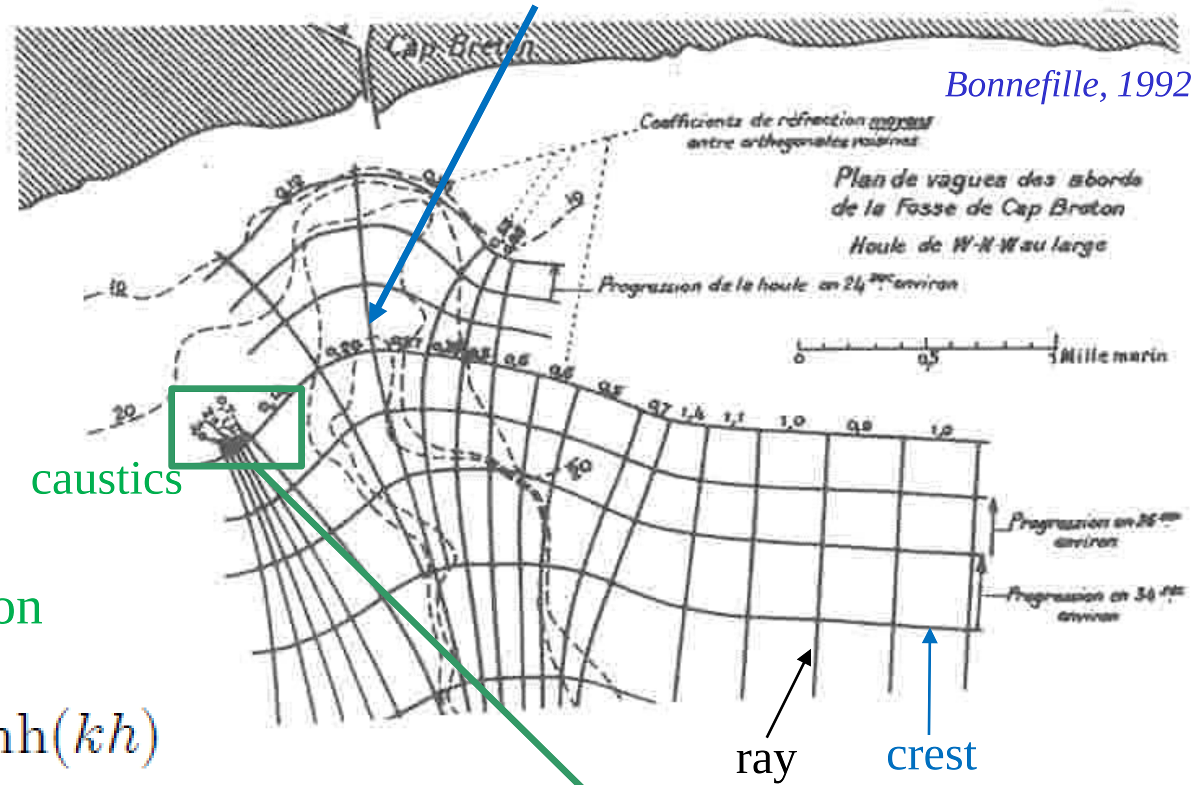
Refraction – diffraction

Wave celerity C



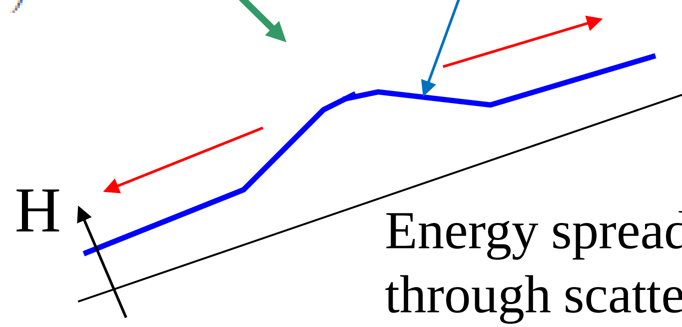
Canyon of Capbreton, France

Bonnefille, 1992



Wave celerity C=ω/k depends on

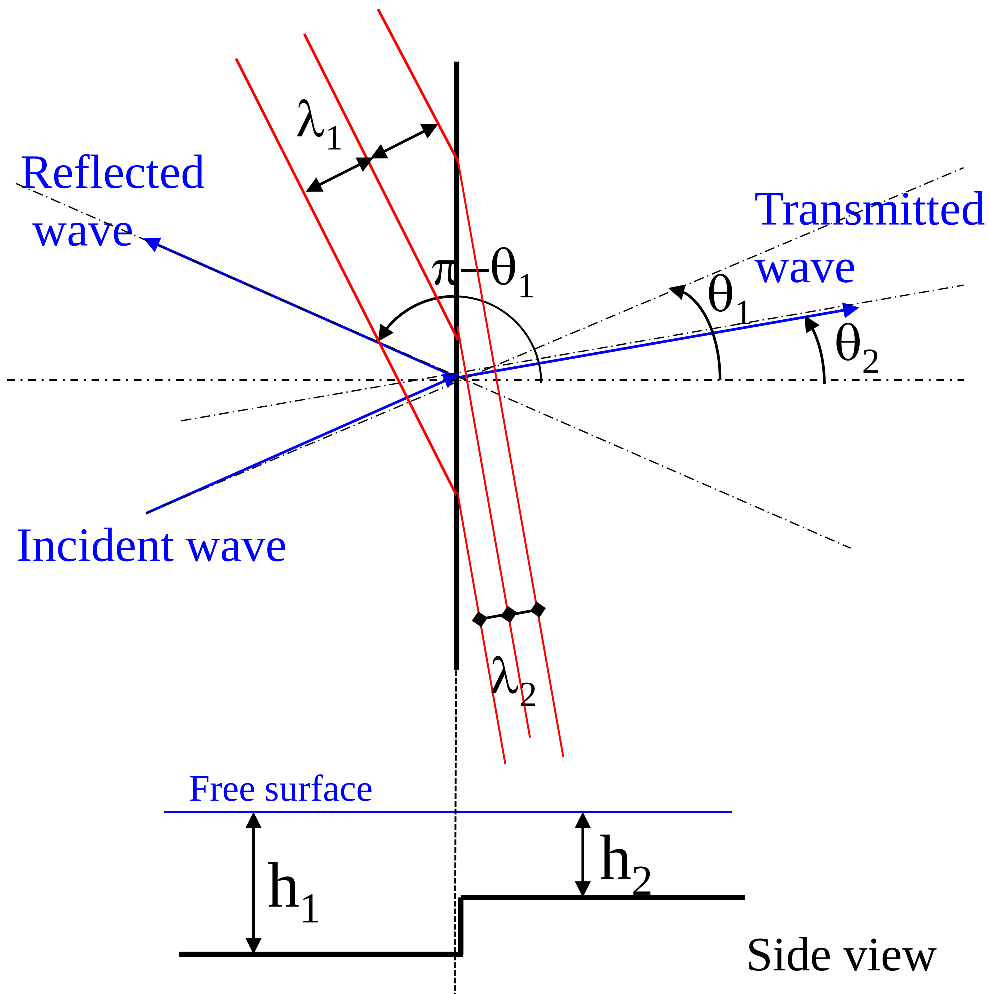
- bathymetry : $\omega^2 = gk\tanh(kh)$
- Currents : $(\omega - Uk)^2 = \sigma^2 = gk\tanh(kh)$
- Porous media : $Z\omega^2 = igk\tanh(kh)$



Energy spreading through scattering

Refraction – Descartes – Snell's Law

1D step effect



Descartes-Snell's Law

$$\frac{\sin\theta_1}{C_1} = \frac{\sin\theta_2}{C_2} \text{ with } C_1 = \frac{\omega}{k_1}, C_2 = \frac{\omega}{k_2}$$

Slowly varying depth
(WKB approximation, mild slope)

$$\eta = a\exp(iS)$$

where $S(\vec{x}, t, \vec{k}, \omega)$

in the plane wave approximation

$$\vec{k} = -\vec{\nabla}_h S \text{ and } \omega = \frac{\partial S}{\partial t}$$

$$\vec{k}(k_1 = k\cos\theta; k_2 = k\sin\theta)$$

and

$$S = \omega t - k_1 x - k_2 y$$

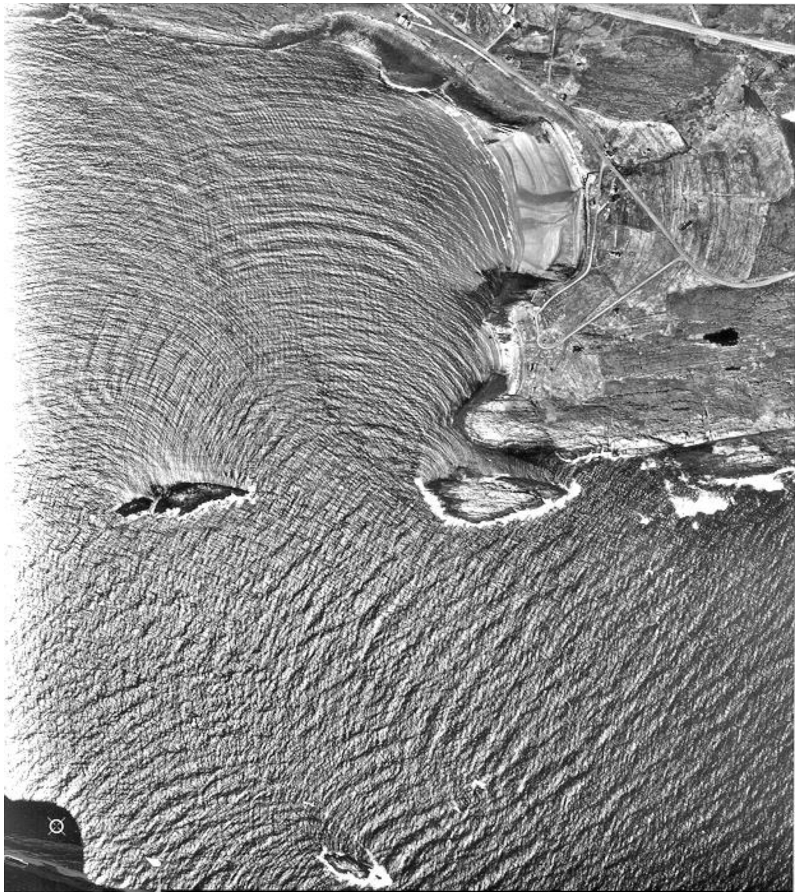
Eikonal equation

$$\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 = \left(\vec{\nabla}_h S\right)^2 = k^2$$

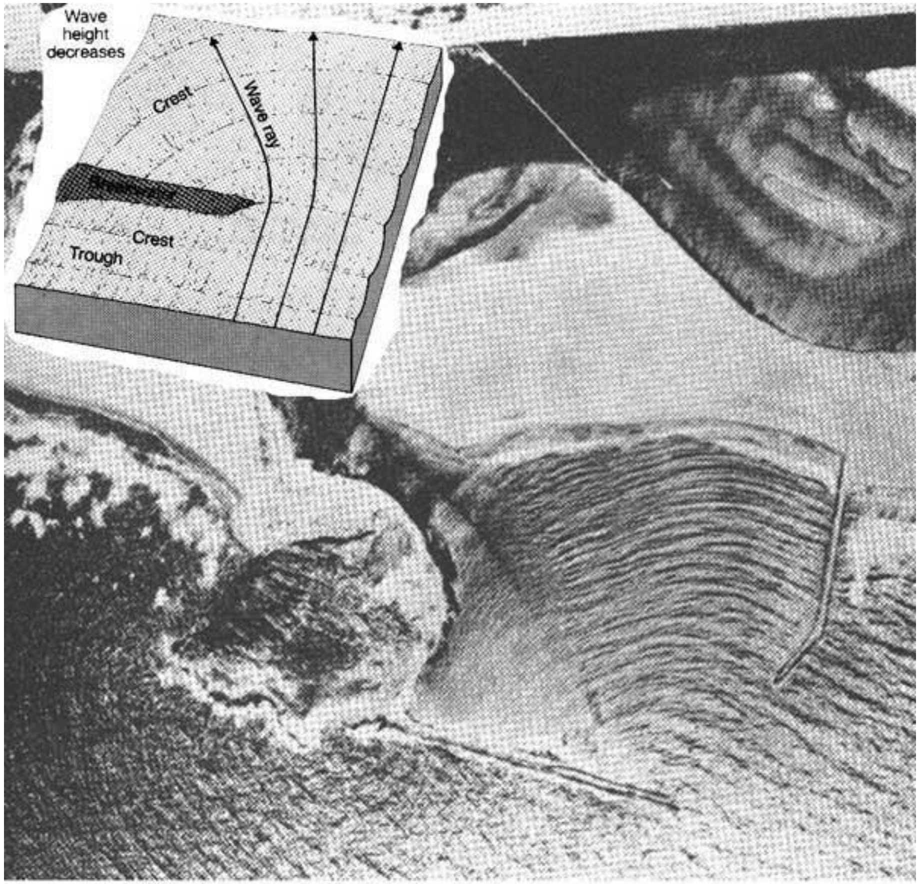
since $\overrightarrow{\mathrm{curl}}(\vec{k}) = \vec{0}$:

$$\frac{\partial(k\sin\theta)}{\partial x} - \frac{\partial(k\cos\theta)}{\partial y} = 0$$

Diffraction



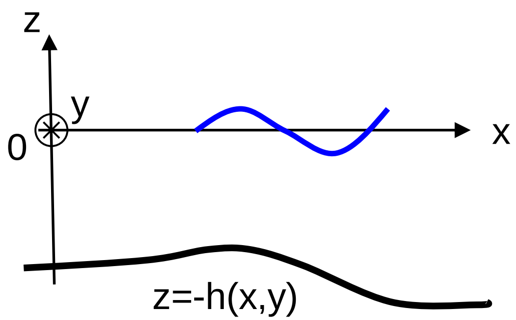
Diffraction due to rapid bathymetric changes



Diffraction occurs behind the breakwater.

Diffraction from abrupt structures (dikes, jetties)

Refraction-diffraction due to « rapid » bathymetric changes
Berkhoff equation



Velocity potential : $\Phi(x,y,z,t) = \phi(x,y,z)e^{i\omega t}$

$$\phi(x,y,z) = \frac{ig}{\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)}\eta(x,y)$$

Hypothesis (mild slope) : $\mu = \frac{\nabla h}{kh} \ll 1$

Conservation equation and boundary conditions :

$$\begin{aligned} \frac{\partial^2 \phi}{\partial z^2} &= -\nabla_h^2 \phi \text{ for } 0 \geqslant z \geqslant -h(x,y) \\ \frac{\partial \phi}{\partial z} - \frac{\omega}{g}\phi &= 0 \text{ for } z = 0 \\ \frac{\partial \Phi}{\partial z} &= \vec{\nabla}_h h \vec{\nabla}_h \phi \text{ for } z = -h(x,y) \end{aligned}$$

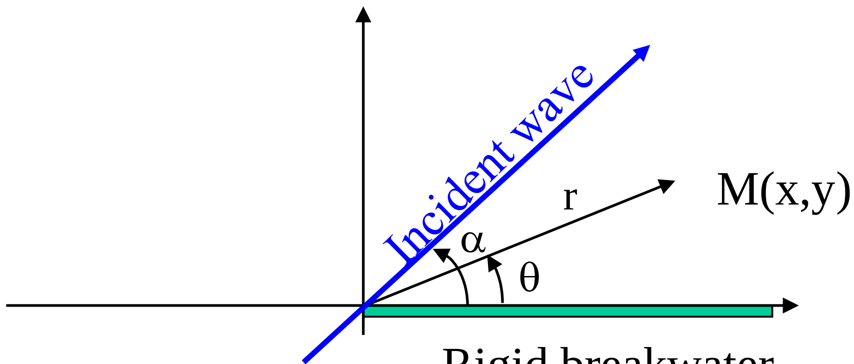
Weak formulation of the Laplace's equation :

$$\int_{-h}^{0} \left(k^2 f\phi + \nabla_h^2 \phi f\right) dz = \left[\left(\nabla_h h \nabla_h \phi\right) f\right]_{z=-h}$$ with the Green function $f = \cosh[k(z+h)]$

Mei, 1983 ⟶ $\vec{\nabla}_h.\left(CC_g\vec{\nabla}_h\eta\right) + k^2 CC_g\eta = 0$ Berkhoff equation

Diffraction due to vertical walls : **Diffraction behind a breakwater**

Wave of incidence α :



Velocity potential :

$$\Phi(x,y,z,t) = \frac{a\omega}{k}\frac{\cosh[k(z+h)]}{\sinh(kh)}F(x,y)e^{i\omega t}$$

F verifies the Helmotz equation : $\nabla_h^2 F + k^2 F = 0$

Solution (polar coordinates) :

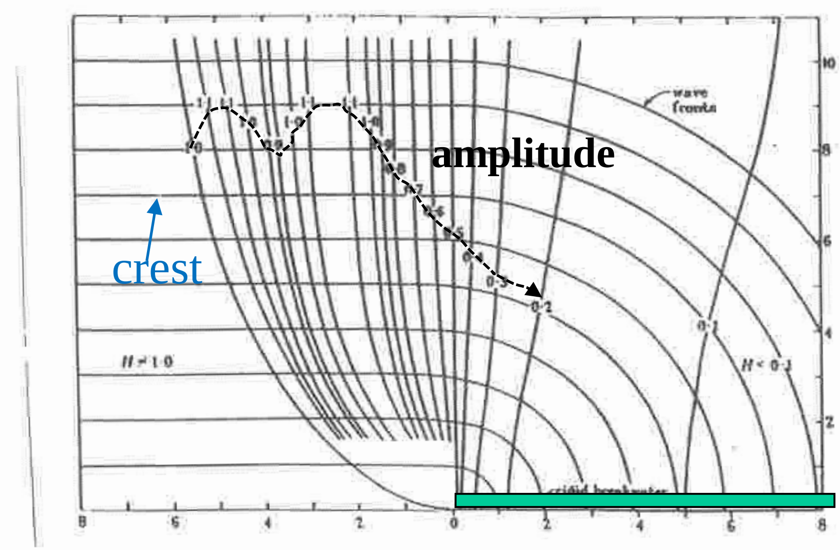
$$F(r,\theta) = I\left(-\sqrt{\frac{4kr}{\pi}}\sin\frac{\alpha-\theta}{2}\right)e^{ikr\cos(\alpha-\theta)}$$

$$+I\left(-\sqrt{\frac{4kr}{\pi}}\sin\frac{\alpha+\theta}{2}\right)e^{-ikr\cos(\alpha+\theta)}$$

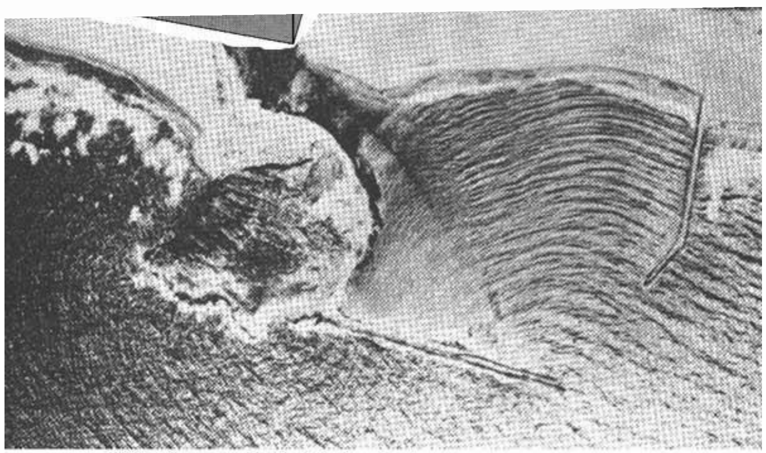
with $I(u) = \frac{1}{2}(1+i)\int_{-\infty}^{u} e^{-i\frac{\pi u'^2}{2}}du'$

Penney and Price, 1952, Horikawa, 1988

Normal incidence (α=π/2) :



Rigid breakwater



Wave diffraction behind the breakwater

Propagation in the presence of vertical walls : Expression of the velocity potentials

$$\Phi(x,y,z,t) = \cosh[k(z+h)]\phi(x,y)e^{i\omega t}$$
$$= \cosh[k(z+h)]\sum_{n=0}^{\infty}\left[A_n^- e^{-ik_{xn}x} + A_n^+ e^{+ik_{xn}(x-L)}\right]\psi_n(y)e^{i\omega t}$$

with $\psi_n(y) = \cos k_{yn}(y - d_m)$

where $k_{yn} = \frac{n\pi}{d_M - d_m}$

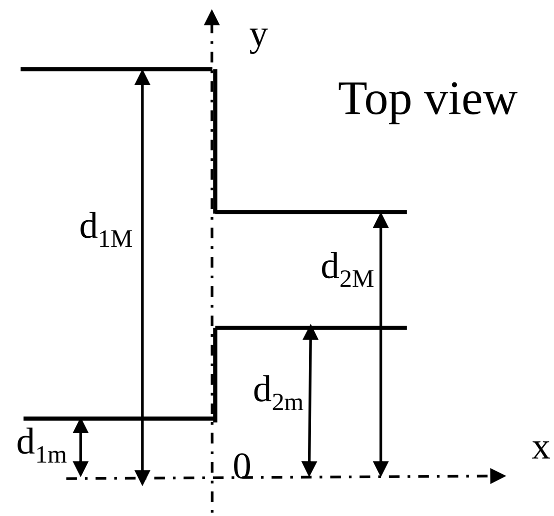
h = constant water depth

Since $\nabla^2\Phi = 0,\ k_{xn} = \left(k^2 - k_{yn}^2\right)^{\frac{1}{2}}$

$$n_{prop} = 1 + Int\left[\frac{kd}{\pi}\right]$$

$n < n_{prop}$ Propagating modes of direction $\theta_n = \pm \text{atan}\frac{|k_{yn}|}{|k_{xn}|}$

$n > n_{prop}$ Evanescent modes along the x-axis

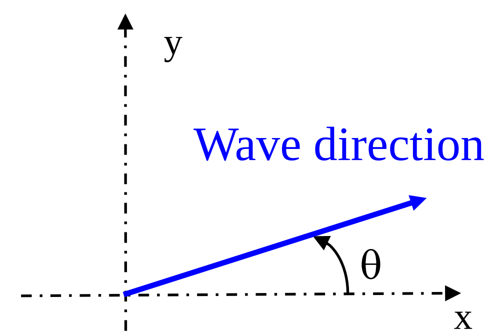


Integral matching method, at a width discontinuity:

Pressure: $$\int_{d_{m2}}^{d_{M2}} \phi_1.\psi_{2,n}dy = \int_{d_{m2}}^{d_{M2}} \phi_2.\psi_{2,n}dy$$

Velocity: $$\int_{d_{m1}}^{d_{M1}} \frac{\partial\phi_1}{\partial x}.\psi_{1,n}dy = \int_{d_{m2}}^{d_{M2}} \frac{\partial\phi_2}{\partial x}.\psi_{1,n}dy$$

for $n = 0, .., P$



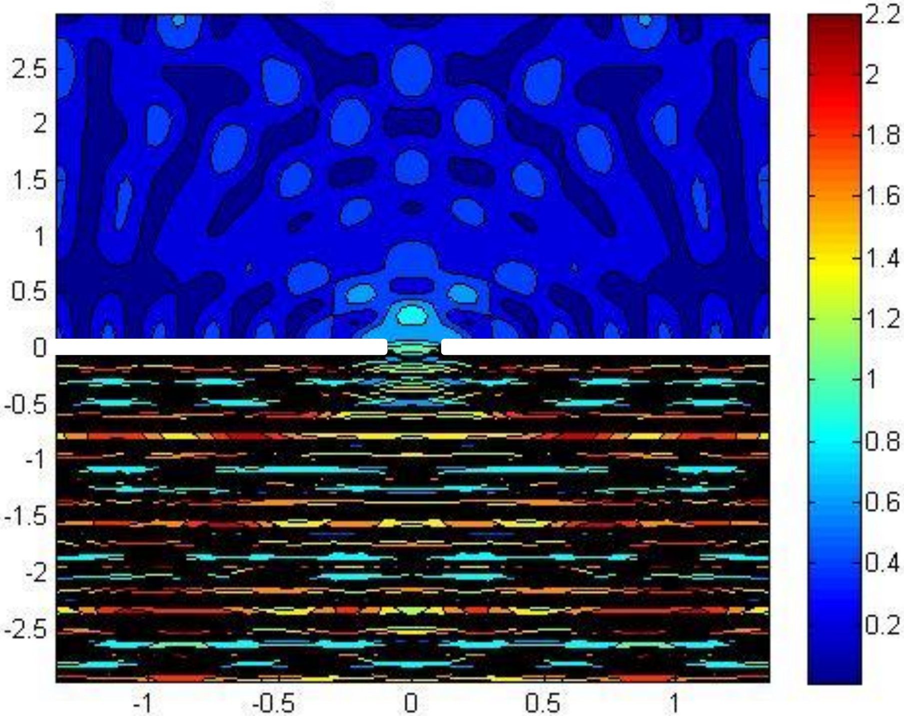
Rey et al, 2018

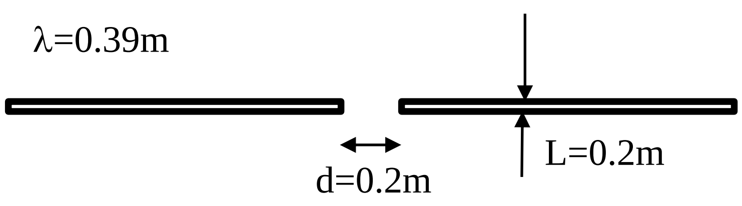
Propagation in the presence of vertical walls : examples

Top views

Sub-wavelength hole

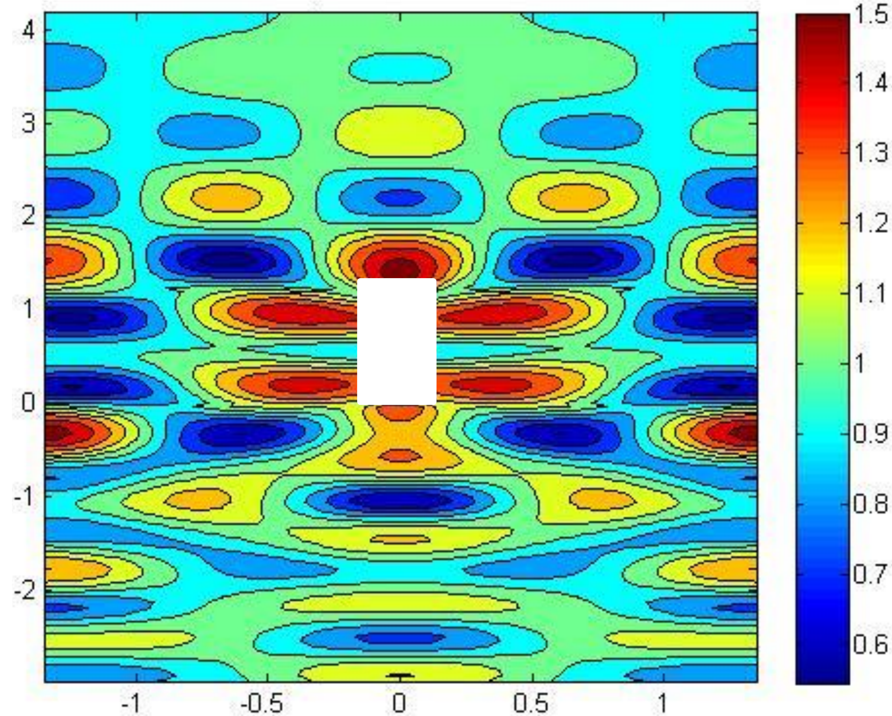
dimensionless amplitude with respect to the incoming wave



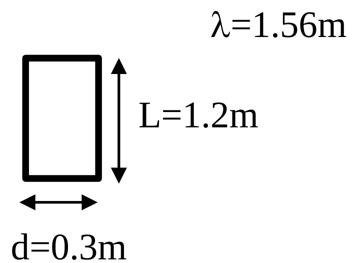


Rectangular emerging structure

dimensionless amplitude with respect to the incoming wave

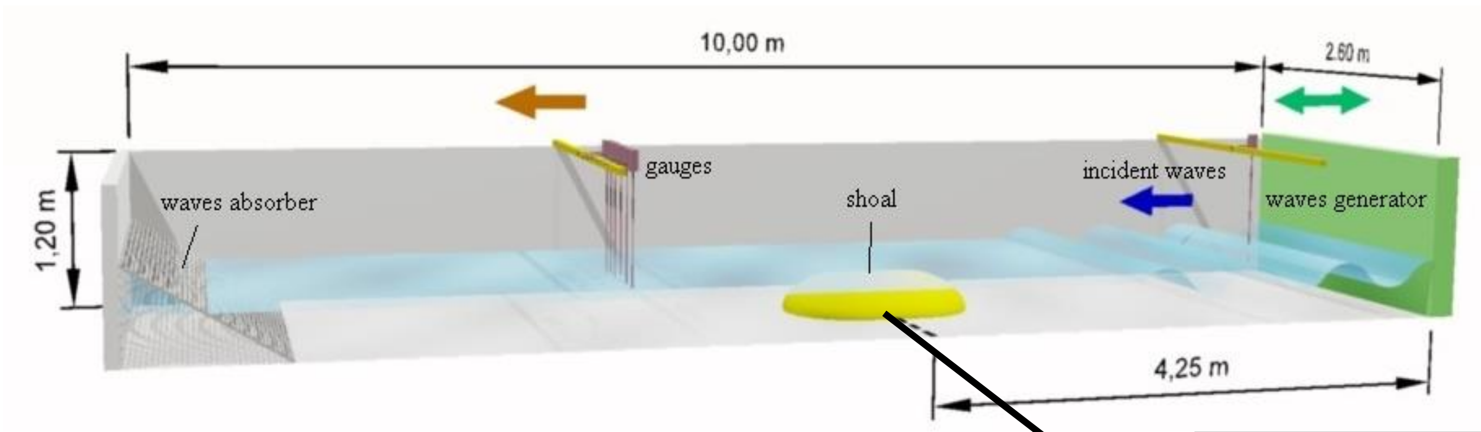


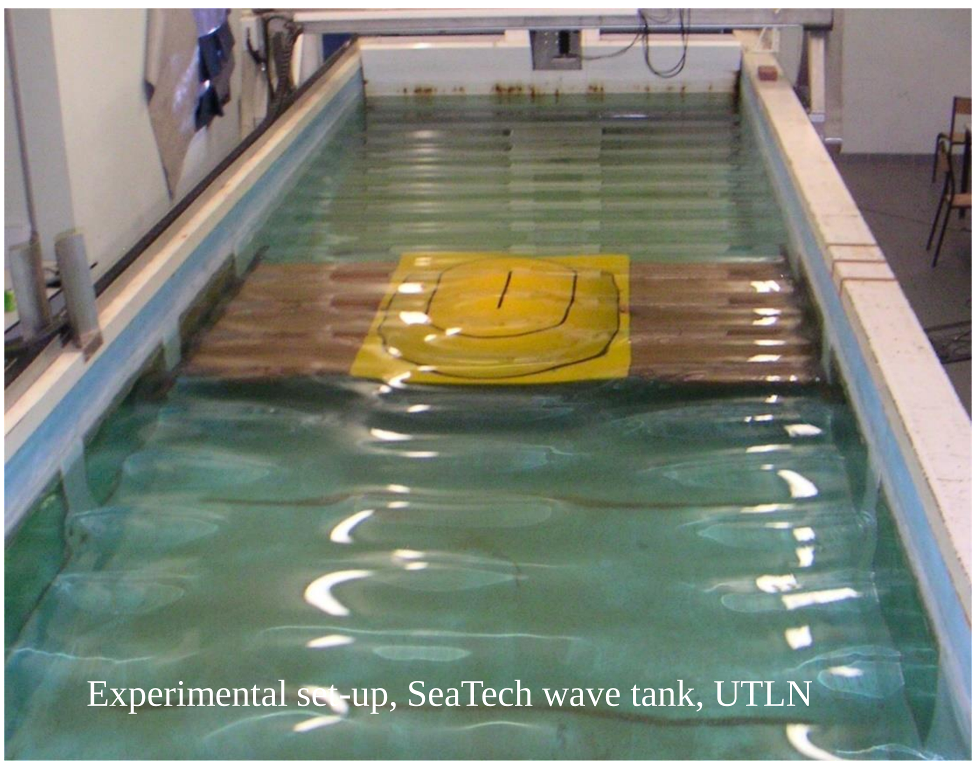
Incoming wave direction

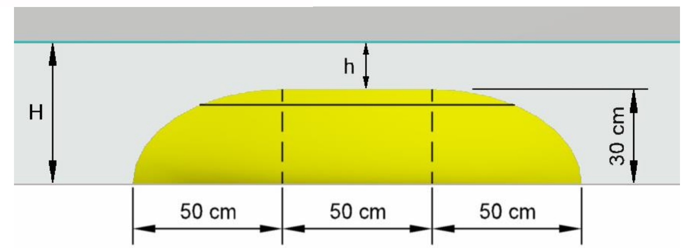


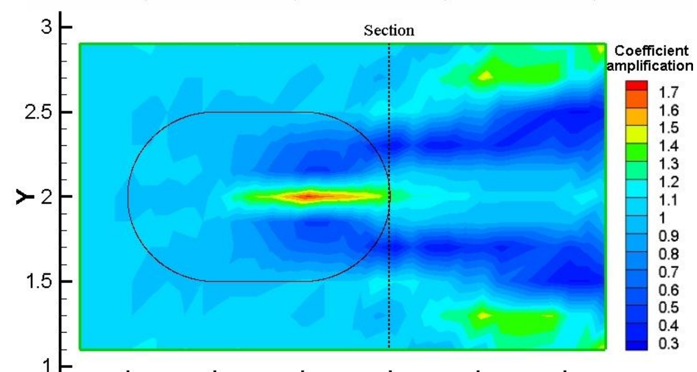
➡ either spreading or focusing effects

Propagation in inhomogeneous media : wave focusing above a shoal




Experimental set-up, SeaTech wave tank, UTLN

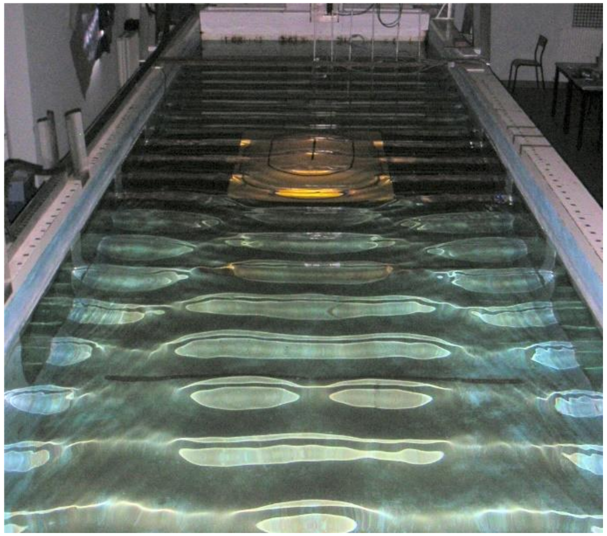


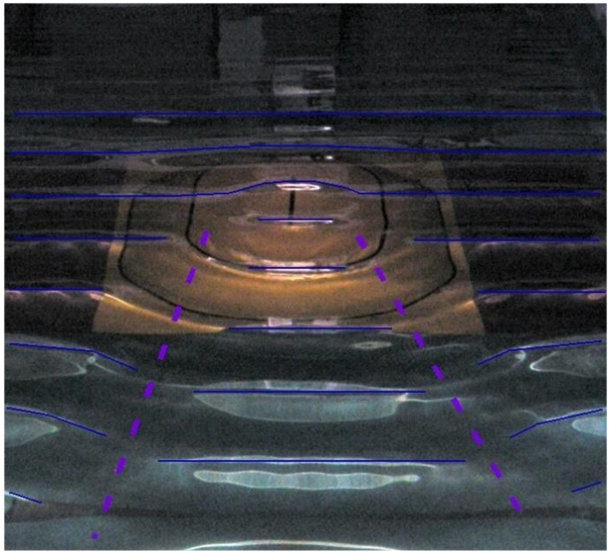


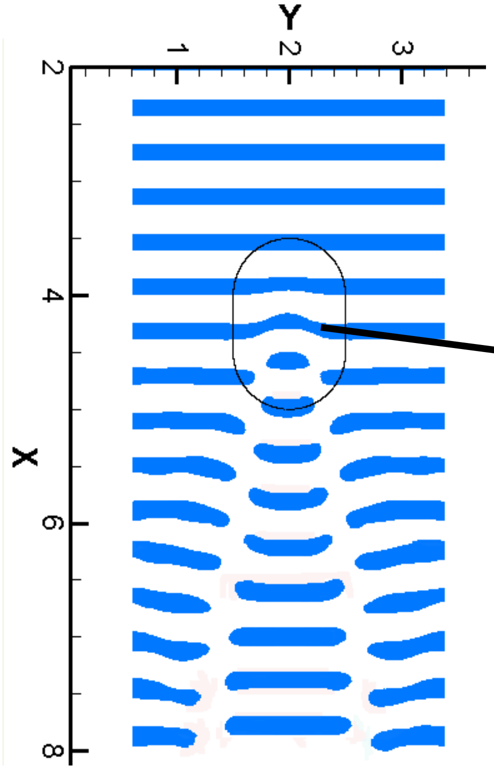
Period	Maximum of amplification	Distance from the end of the mound (m)
T=0.3	1	-
T=0.4	1.12	-0.20
T=0.5	1.31	-0.10
T=0.6	1.53	-0.10

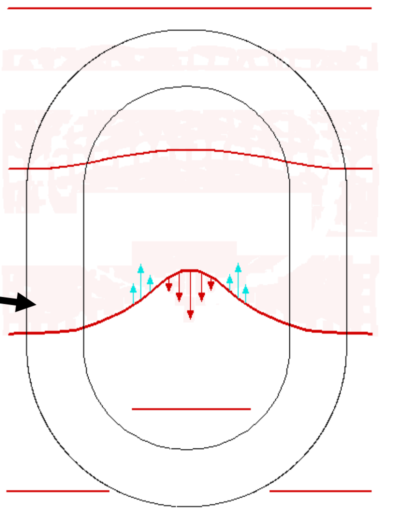
Jarry et al, 2009 ➡ wave focusing especially for longer waves

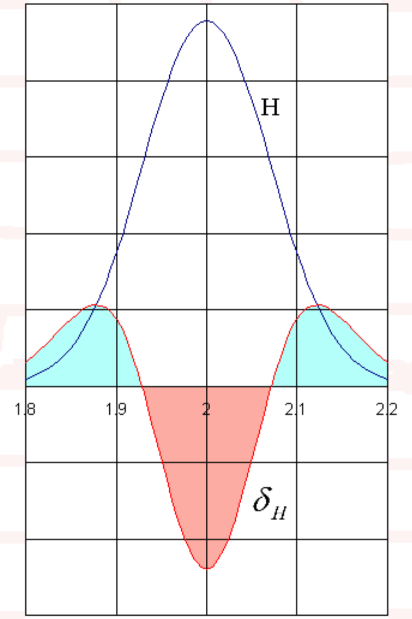
Wave focusing above a shoal : influence of the diffraction term δ_H











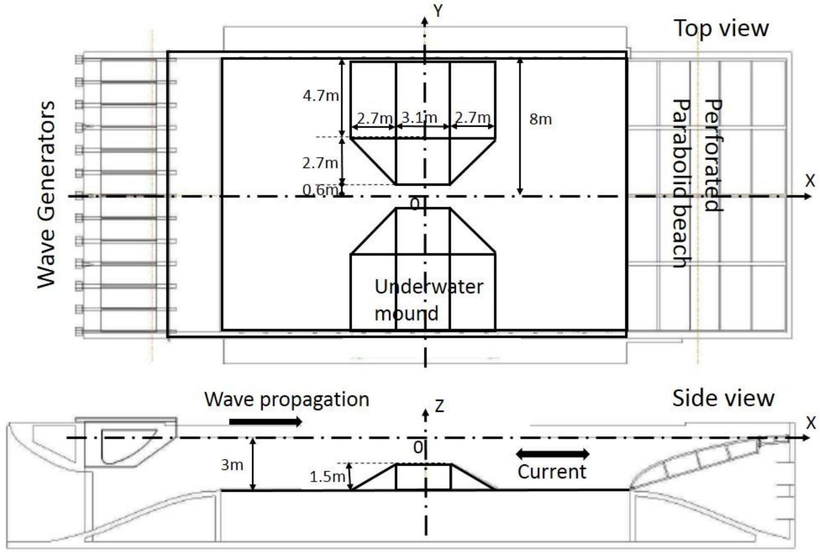
$$H = \hat{H} e^{iS}$$

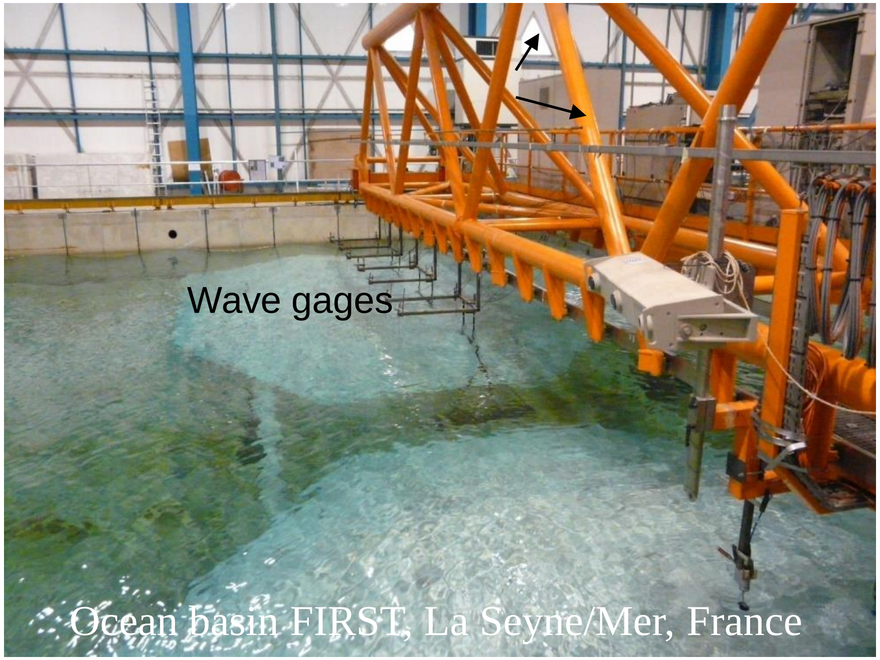
$$\delta_H = \frac{\vec{\nabla}_h \cdot \left(C C_g \hat{H}^2 \vec{\nabla}_h S \right)}{k^2 C C_g \hat{H}}$$

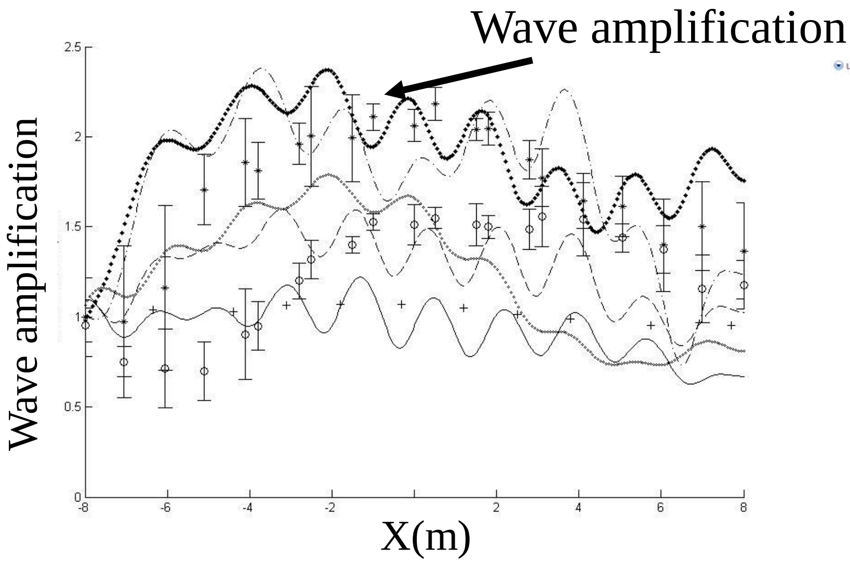
$$\begin{cases} k' &= k\sqrt{1+\delta_H} \\ C' &= C/\sqrt{1+\delta_H} \\ C'_g &= C_g/\sqrt{1+\delta_H} \end{cases}$$

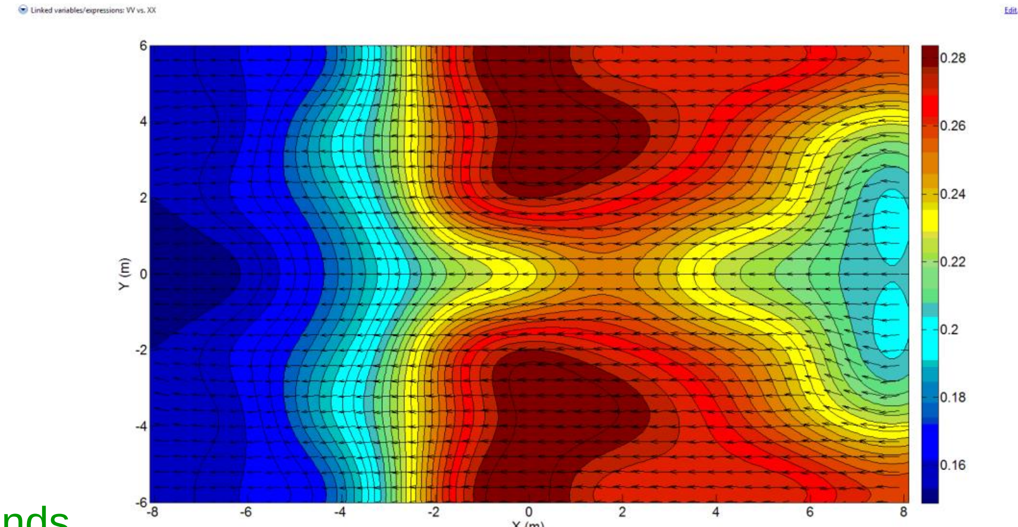
Holtuijsen et al., 2003, Jarry et al, 2009

Propagation in inhomogeneous media : wave focusing in the presence of currents







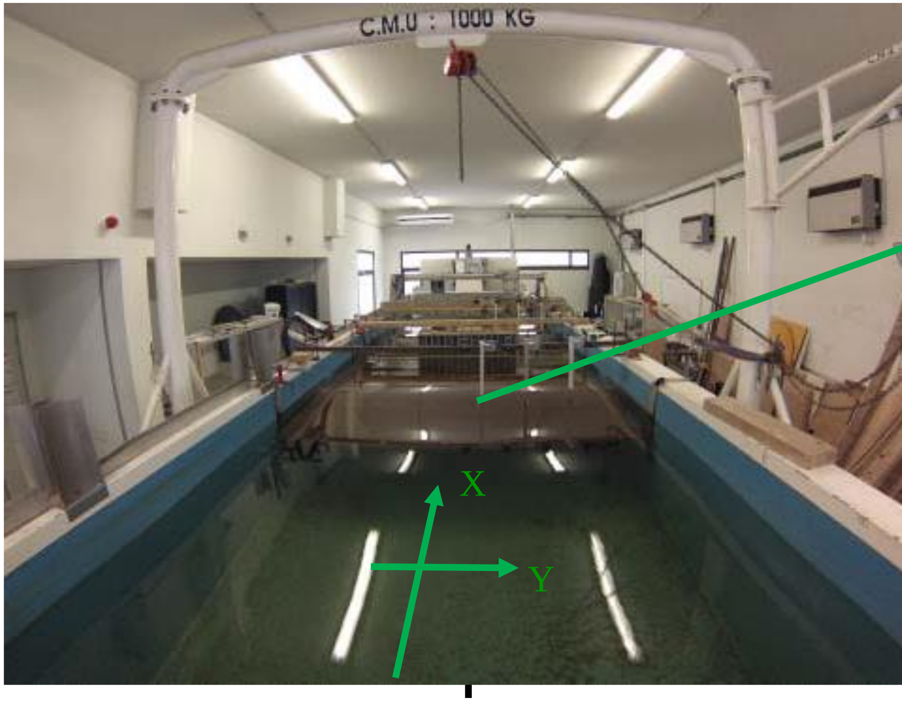


Mean curent field

- current lield forced by the underwater mounds
- Amplification up two twice the incident wave amplitude in deep water conditions for wave-opposing current conditions

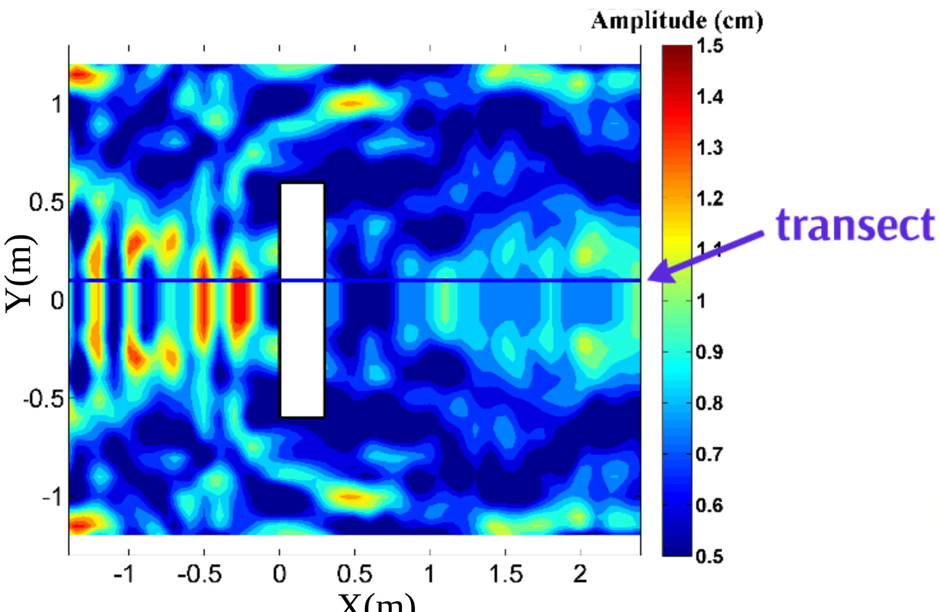
Rey et al, 2014

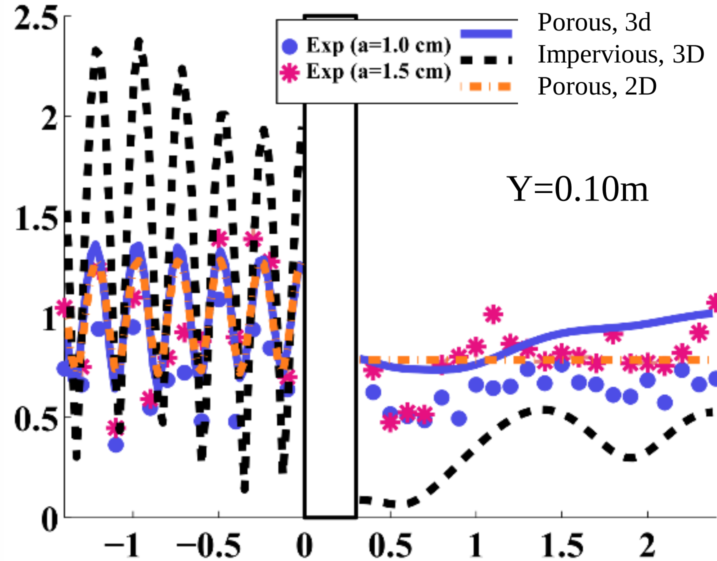
Propagation in inhomogeneous media : wave scattering in the presence porous media



Porous rectangular structure
made of vertical cylinders
L x l = 0.30m x 1.20m

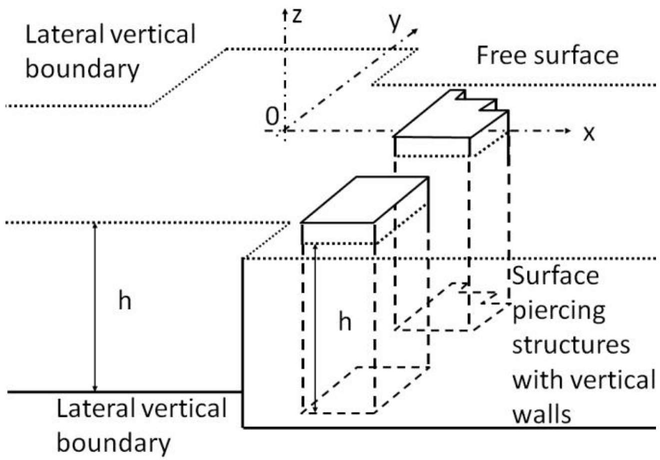
➡ wave scattering:
Reflection upwave, refraction/diffraction
propagation across the porous medium



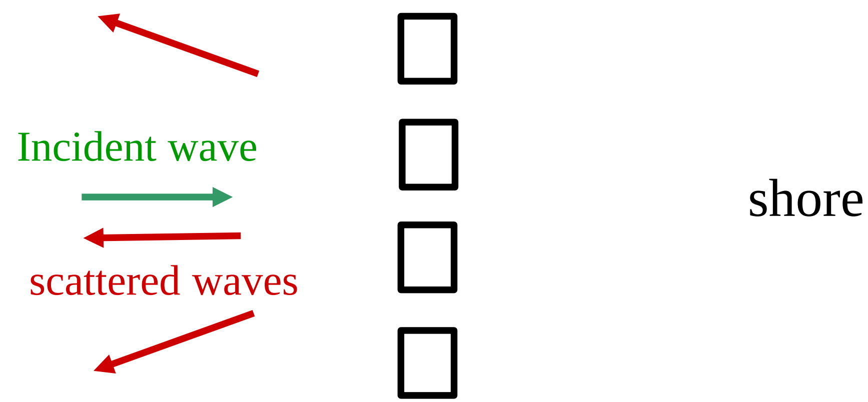


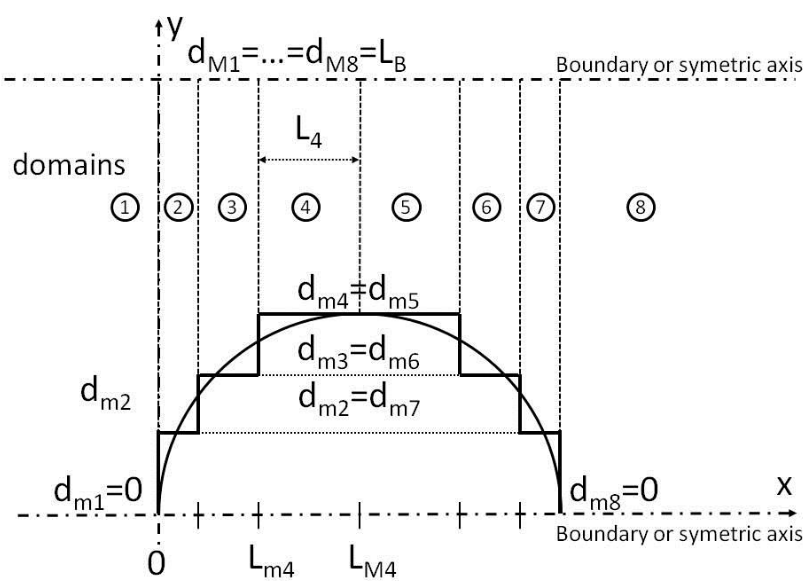
Arnaud, 2016

Propagation in the presence of periodic structures :
wave diffraction and Bragg resonance

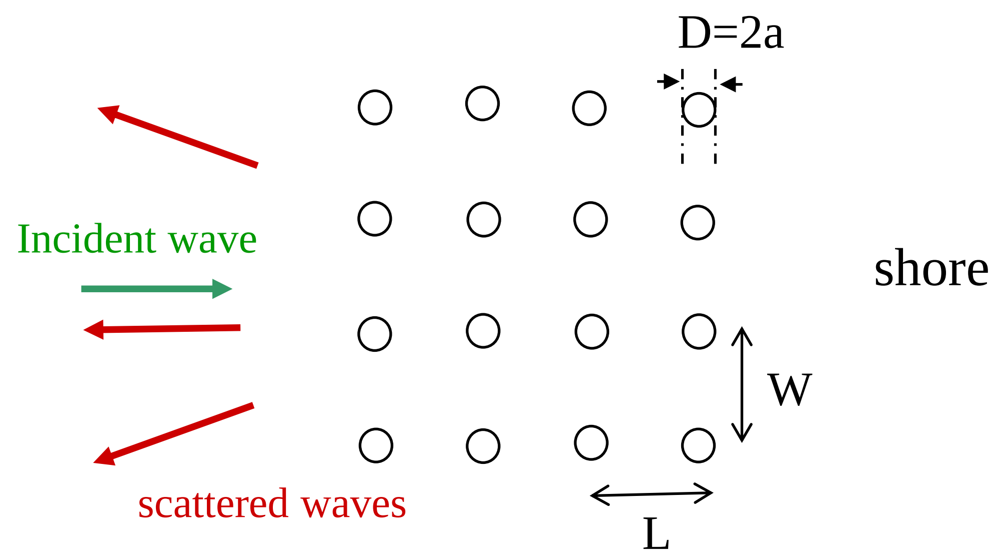


➡ Periodic breakwaters of rectangular shape





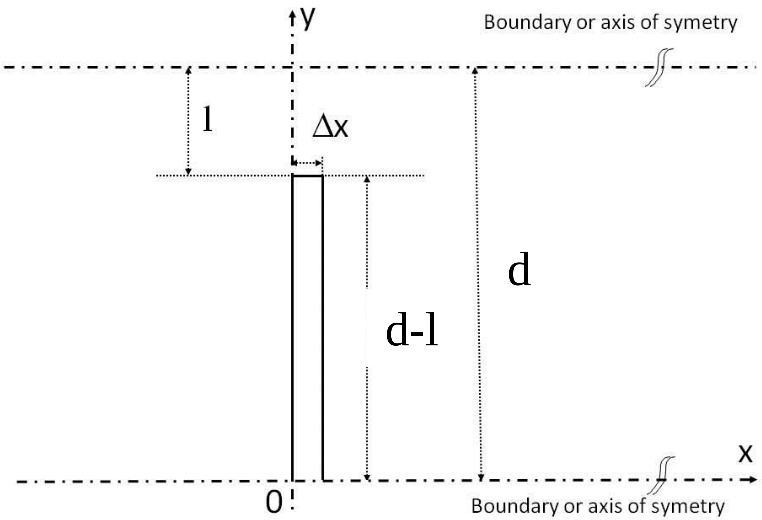
➡ Periodic cylinder arrays



Rey et al, 2018

Periodic breakwaters of rectangular shape

wave scattering



k, wave wavenumber,
Periodicity d of the breakwaters along y-axis

Number of propagating modes:

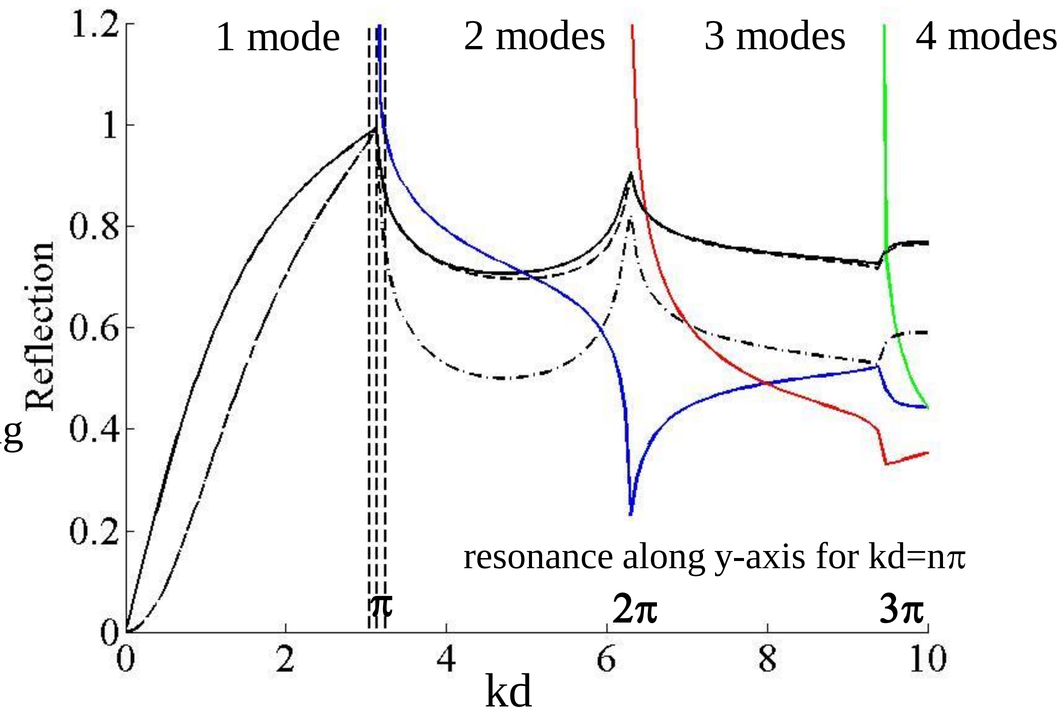
$$n_{prop} = 1 + Int\left[\frac{kd}{\pi}\right]$$

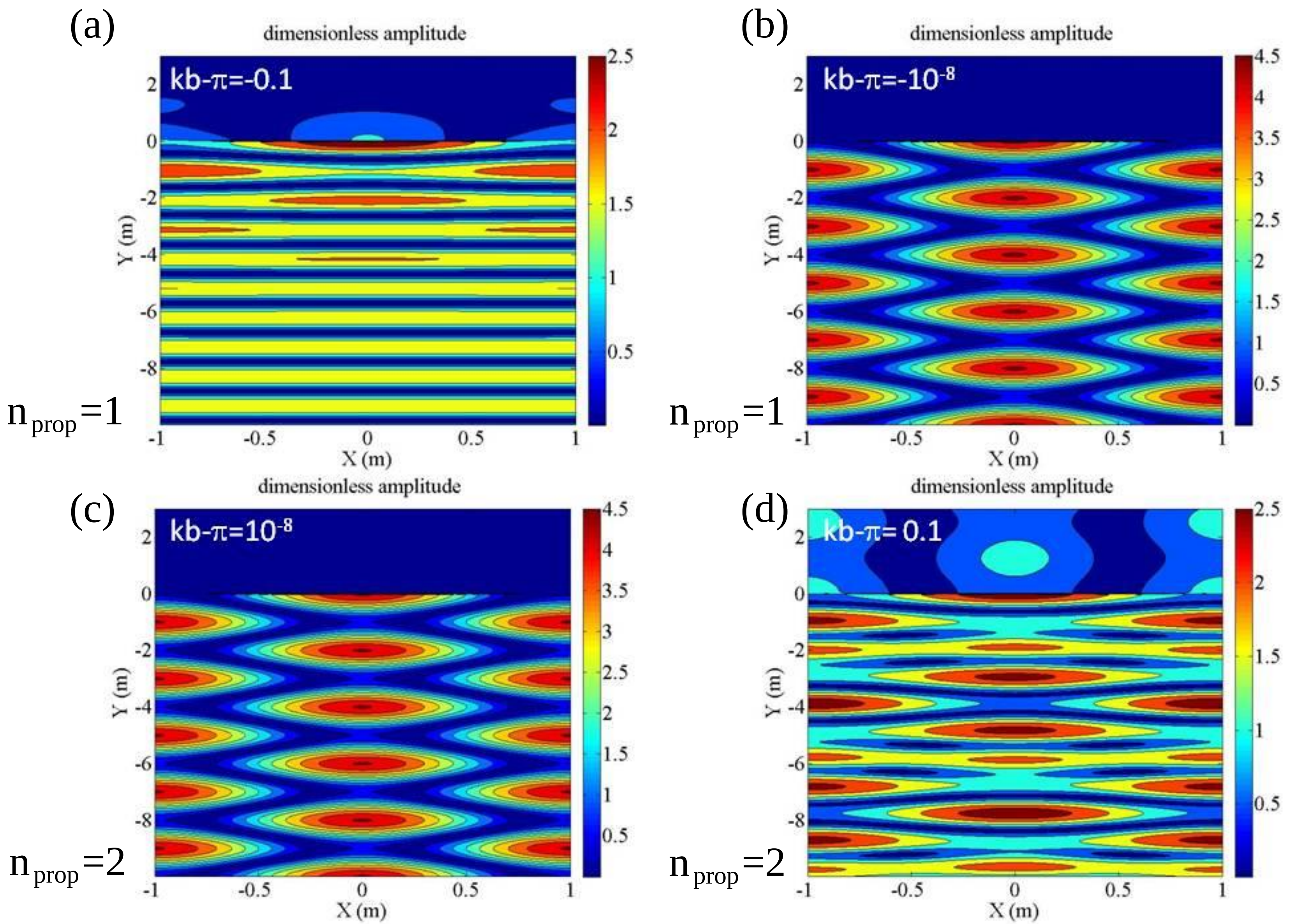
Reflection coefficient versus kb:
(-), 1st mode (n=0); (-) second mode (n=1);
(-) third mode (n=2); (-) fourth mode (n=3);

Relative reflected energy flux versus kb :
(-.-),1st mode (n=0), (- -), total energy. Vertical dashed lines correspond to the locations of the frequency cases presented in the following

Energy flux conservation:

$$1 = R_0^2 + T_0^2 + \frac{1}{2}\sum_{n=1}^{n_{prop}-1}\frac{k_{xn}^{(0)}}{k}\left[R_n^2 + T_n^2\right]$$

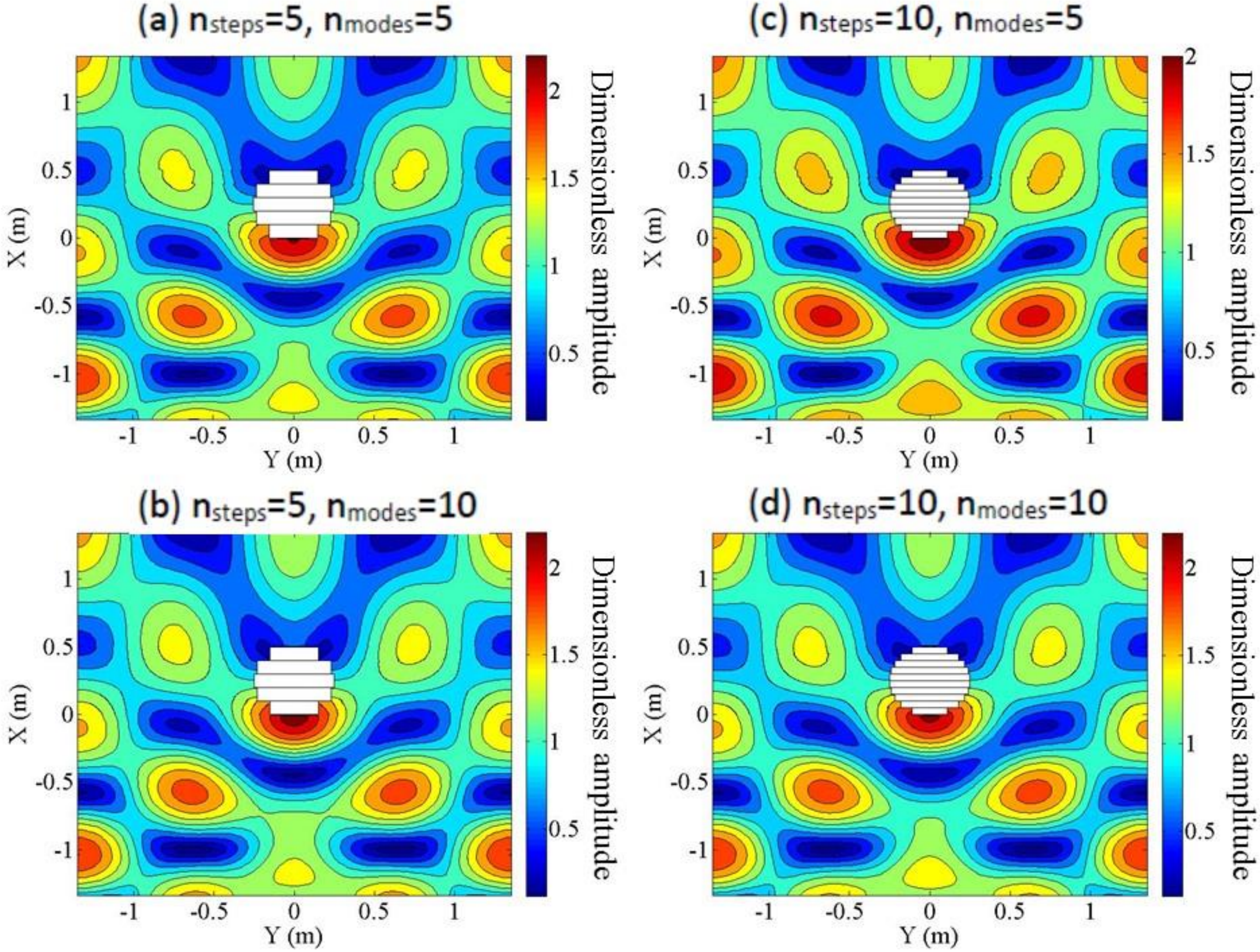




Relative amplitude field relative to the incoming wave amplitude,
(a) $kb - \pi = -0.1$, (b) $kb - \pi = -10^{-8}$, (c) $kb - \pi = 10^{-8}$, (d) $kb - \pi = 0.1$

Propagation in the presence of periodic cylinders :

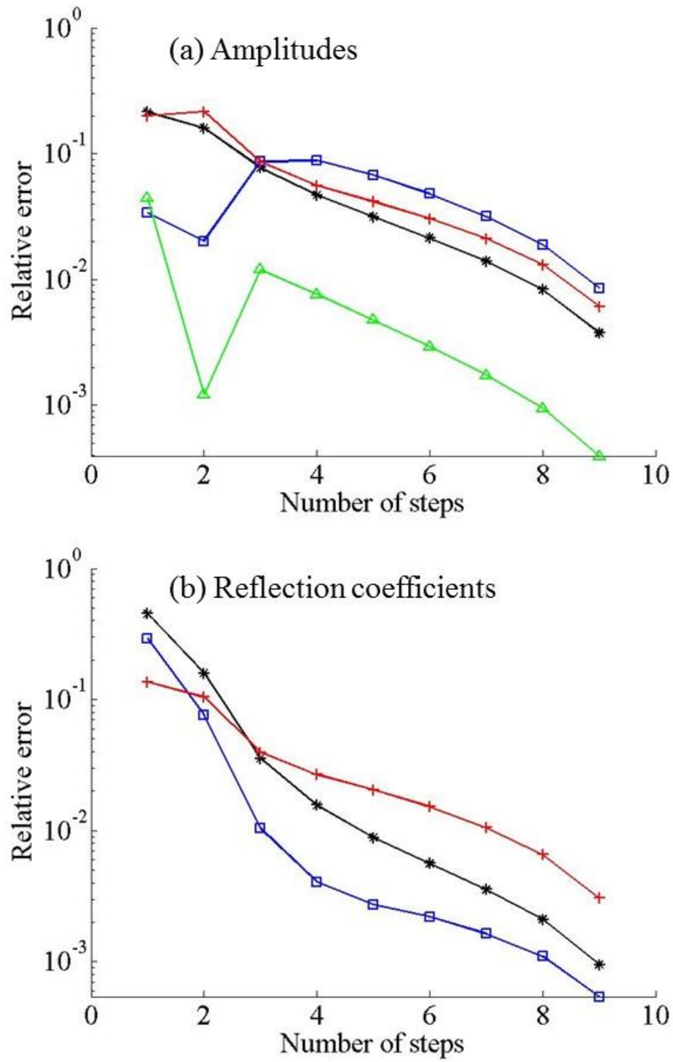
Convergence with respect to the numbers of modes and steps



Relative surface wave amplitude field (a), $n_{steps} = 5$, $n_{modes} = 5$;(b), $n_{steps} = 5$, $n_{modes} = 10$;(c), $n_{steps} = 10$, $n_{modes} = 5$;(d), $n_{steps} = 10$, $n_{modes} = 10$

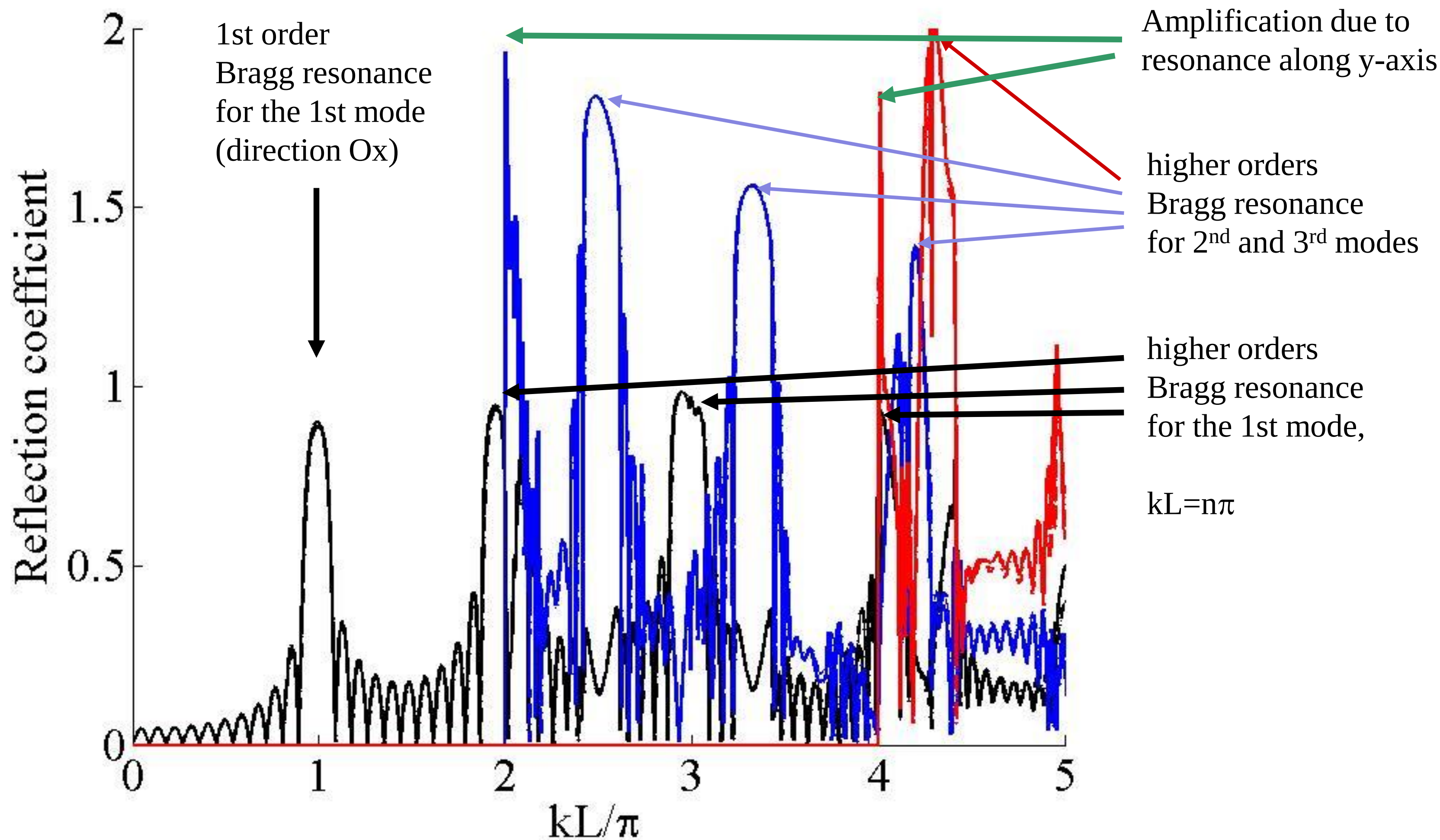
Propagation in the presence of periodic cylinders :

Convergence with respect to the number of steps (for 10 modes)



Relative error (a) of the amplitudes, (b) of the reflection coefficients versus the number of steps n_{steps}. (a), (—*), $x = -0.0135$, $y = 0$; (—□), $x = 0.5085$, $y = 0$; (—+), $x = -0.0135$, $y = 0.6$; (—△), $x = 0.5085$, $y = 0.6$. (b) (—*), first mode; (—□), second mode; (—+), third mode.

Propagation in the presence of periodic cylinders : Bragg resonances

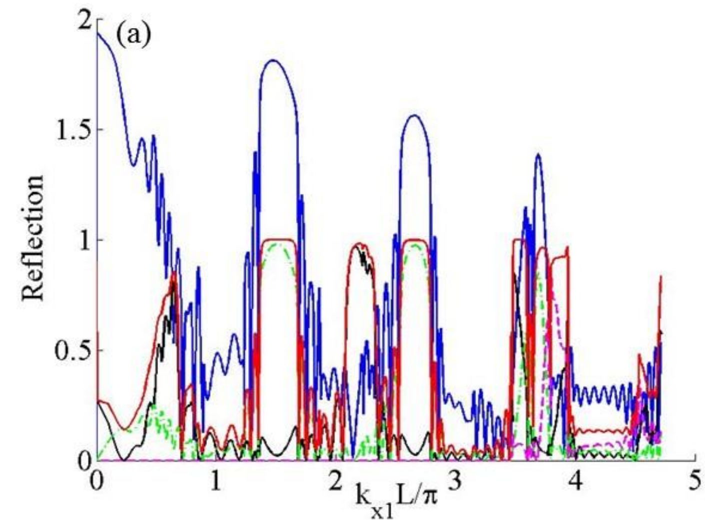


Reflection coefficient in the case of sparse array of 11 cylinders (a/L = 0.10, L/W = 1) versus kL/π: (- -), 1st mode, P = 2 (3 modes); (-), 1st mode, P = 3 (4 modes); (- -), second mode, P = 2 (3 modes), (-), second mode, P = 3 (4 modes); (- -), third mode, P = 2 (3 modes); (-), third mode, P = 3 (4 modes).

Propagation in the presence of periodic cylinders :
Bragg resonances for scattered waves

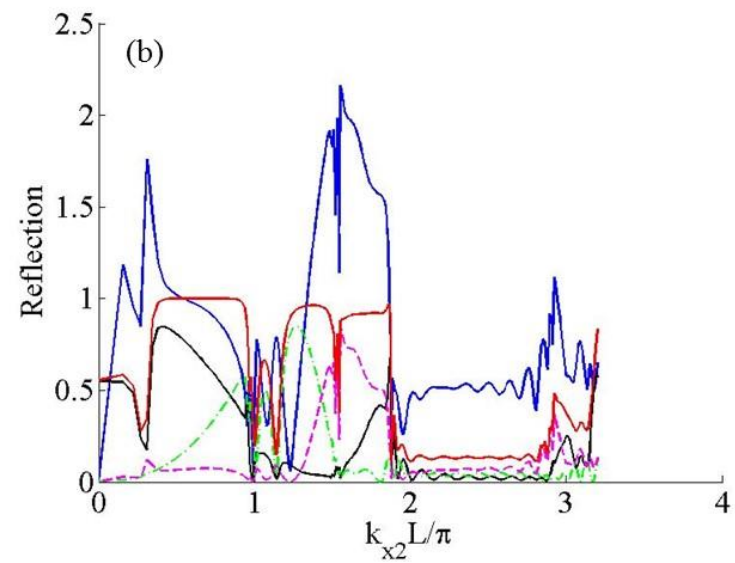
(a) Reflection versus kx1L/π:

(-), reflection coefficient for the second mode;
(-), total reflected energy;
(-), reflected energy for the 1st mode;
(-.-), reflected energy for the second mode;
(- -), reflected energy for the third mode;



(b)Reflection versus kx2L/π:

(-), reflection coefficient for the third mode;
(-), total reflected energy;
(-), reflected energy for the 1st mode;
(-.-), reflected energy for the second mode;
(- -), reflected energyfor the third mode.



Resonance conditions:

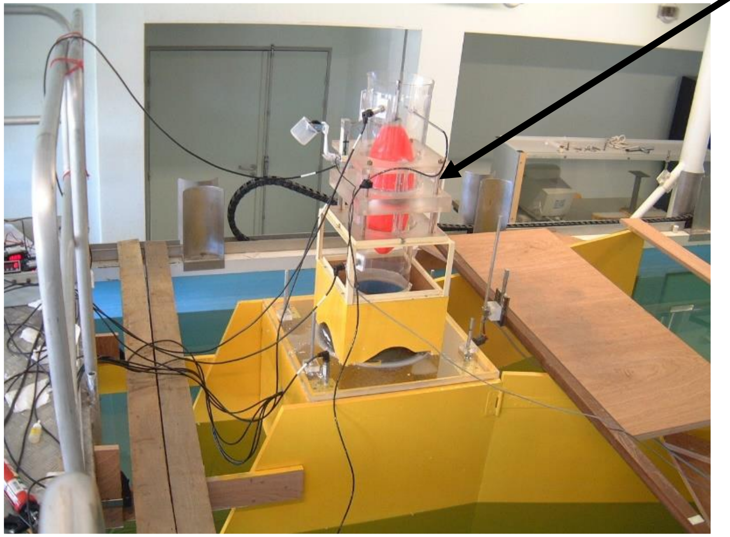
$$k_{xn}L = (2p+1)\pi/2,\ p > 1 \text{ where } n = 1 \text{ for mode 2 and } n = 2 \text{ for mode 3}$$

Application to wave energy device : Oscillating water column (OWC)

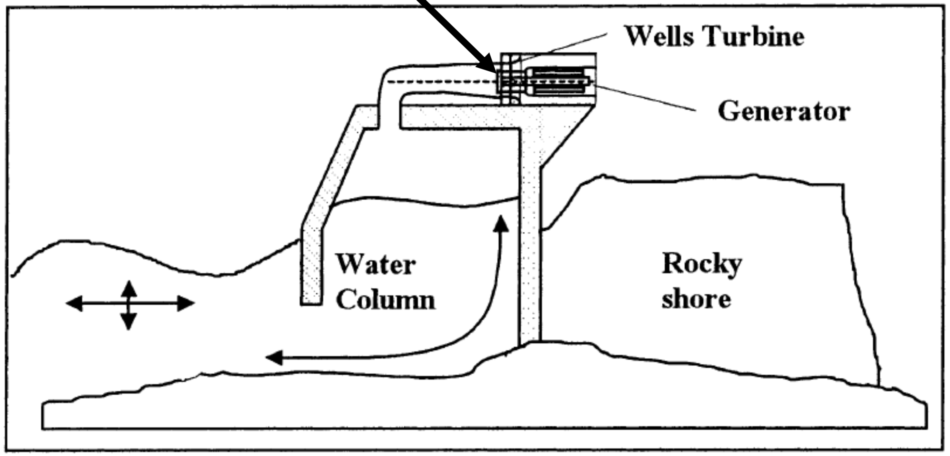
Research project at ISITV, UTLN (years 2000):
How to provide **energy, for punctual needs** in the natural park of Scandola in Corsica, France for public lighting?

Bay of Girolata, Corsica, France

Wells turbine



Prototype, ISITV (now SeaTech), UTLN

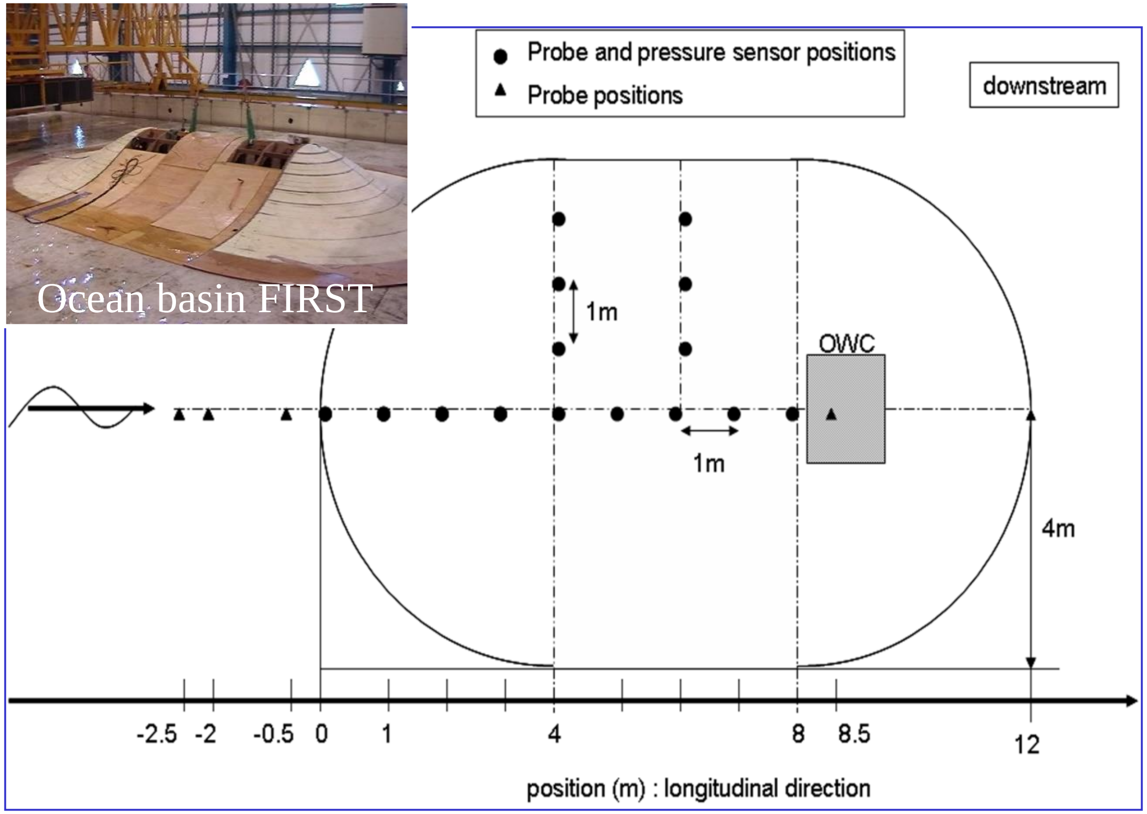


Oscillating water column
(from Delauré and Lewis, 2003)

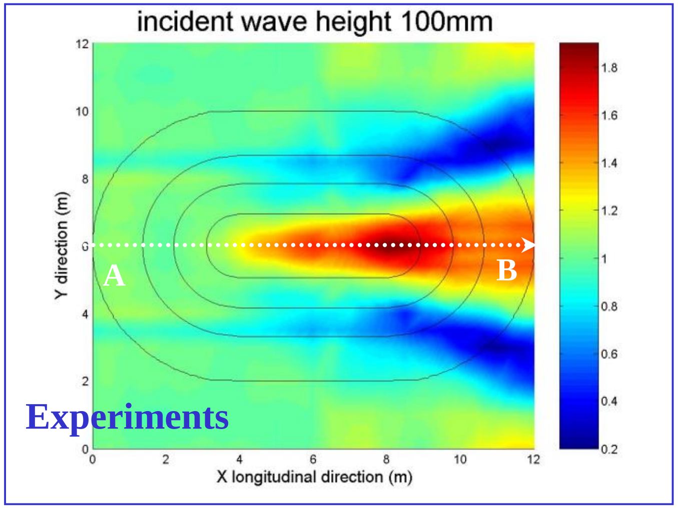
How to increase incoming wave energy?

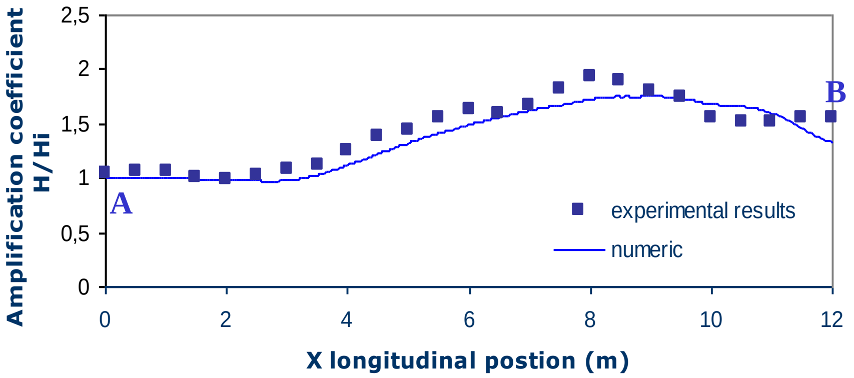
➡ Wave focusing above a shoal

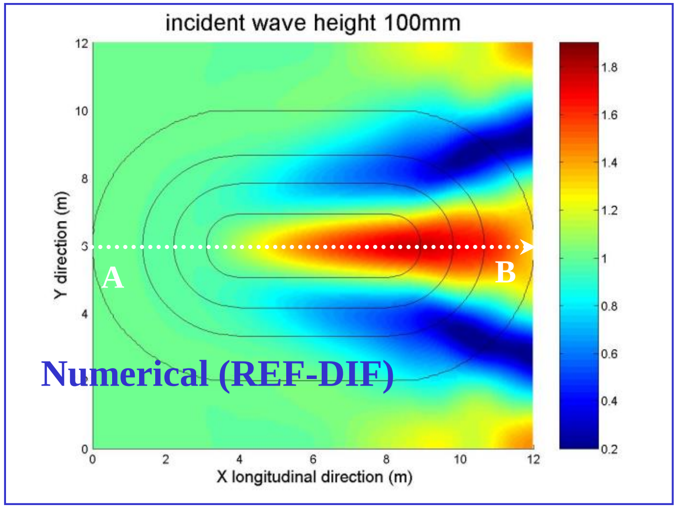
Application to wave energy device : Oscillating water column (OWC)



T=1.7s



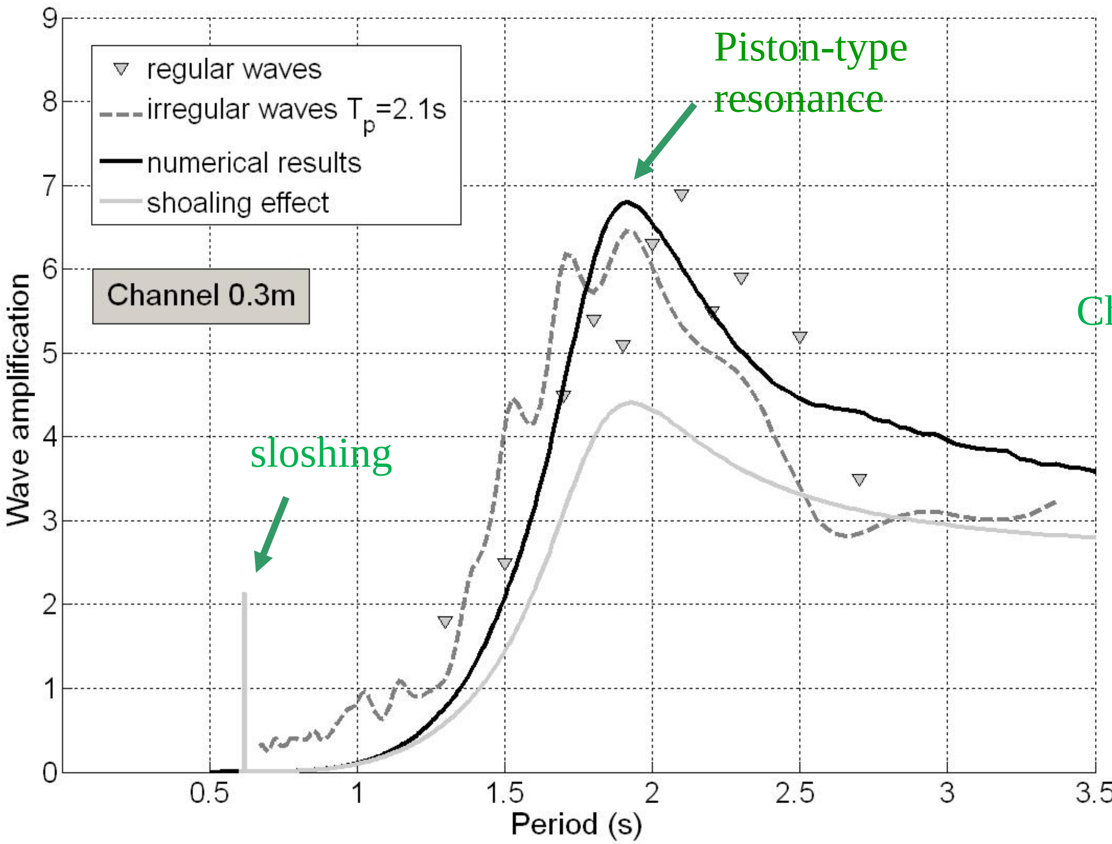


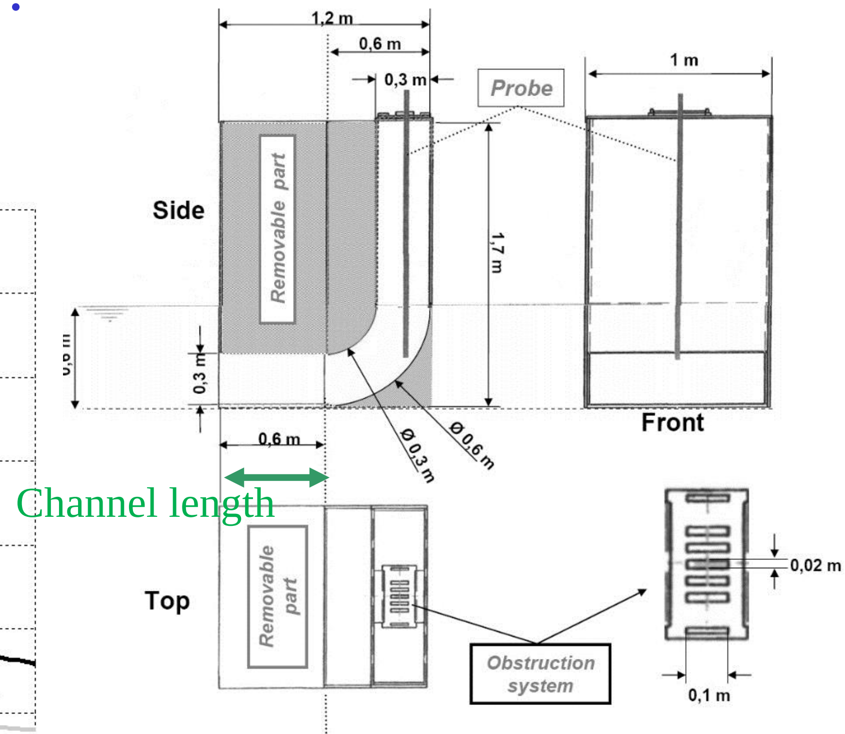


Gouaud et al, 2010

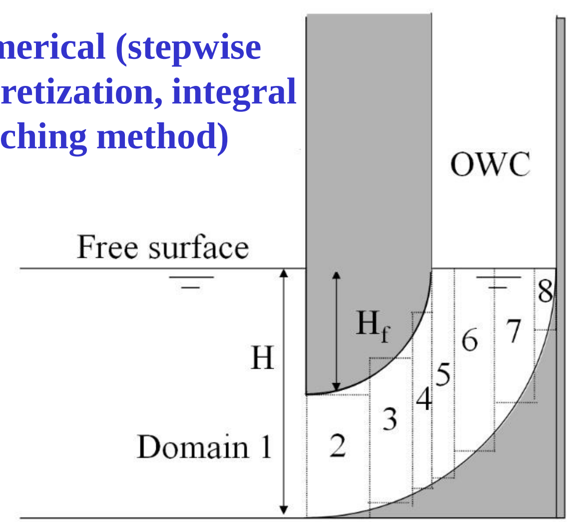
Application to wave energy device :

Wave amplification





Numerical (stepwise discretization, integral matching method)



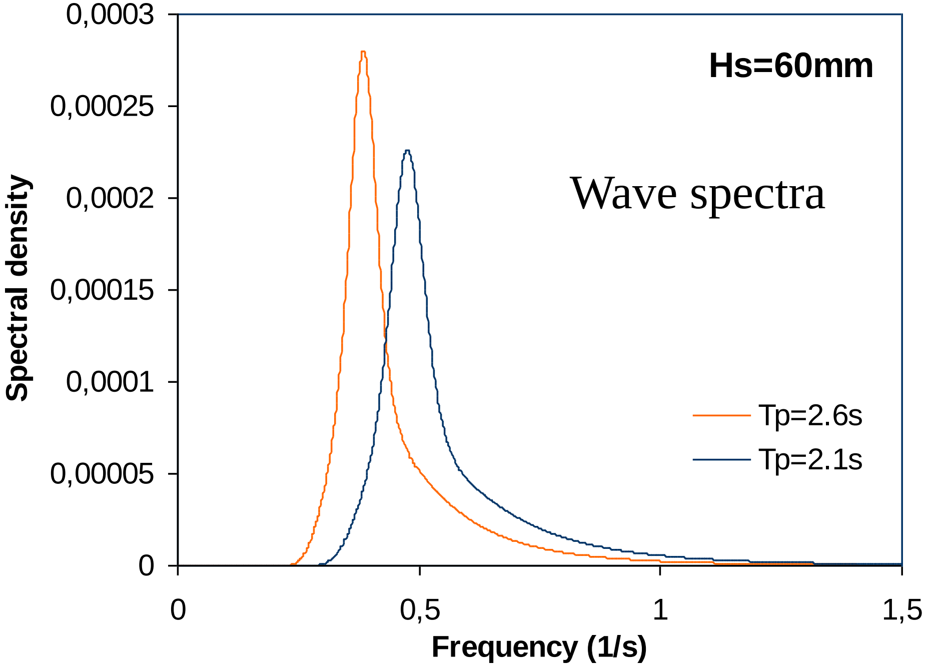
➡ Wave amplification depends on the geometry of the OWC

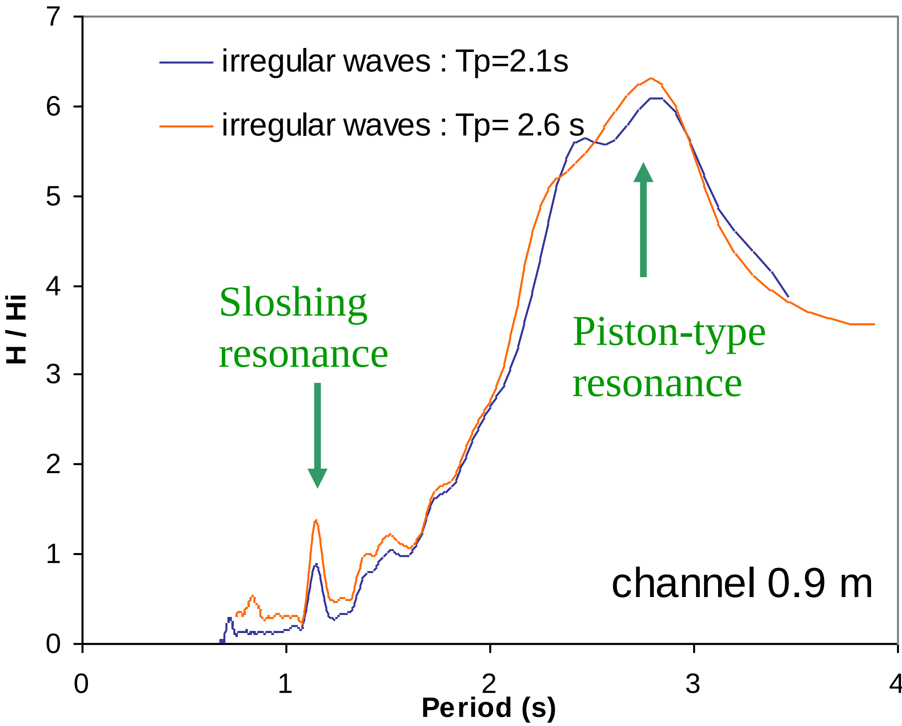
➡ Wave amplification enhanced by the shoal

Gouaud et al, 2010

Wave amplification in the water column

Linear wave behaviour





- **Use of two Jonswap spectra**
- **Both transfer functions are almost identical**

Non-linear effects are rather negligible in that case

Gouaud et al, 2010

Conclusions

→ Inhomogeneous medium : water wave celerity changes
(bathymetry, current, porous medium, presence of surface-piercing structures)

→ Scattering :
2D, reflection, transmission
3D, + refraction, diffraction, focusing or dispersion
Leads to
Interference process due to wave partial reflection at domain boundaries
and Bragg resonance for periodic inhomogeneous media

→ Irregular waves :
interference process less important due to
wave energy spreading on both frequencies and direction of propagation

→ In the 3d Case of constant water depth :
Helmotz equation, analogy to other types of waves as acoustic waves

References

Arnaud, G., Houle à la côte : Propagation, impacts et ouvrages innovants, Ph-D, 3 november 2016, University of Toulon, France (in french).

Arnaud, G., Rey, V., Touboul, J., Sous, D, Molin, B. & Gouaud, F. Wave propagation through dense vertical cylinder arrays:Interference process and specific surface effects on damping, *Applied Ocean Research* 65, 229–237.

Belzons, M., Rey, V. and Guazzelli, E. 1991 Subharmonic Bragg resonance for surface water waves, Europhysics Letters 16, 189-194.

David De Drézigué, O. Sous D. and Rey, V. Wave propagation over a steep bathymetric slope : influence of the bed porosity on the wave phase matching , *Coastal Dynamics 13*, Arcachon, France, 24-28 june 2013.

Delauré, Y and Lewis, 3D hydrodynamic modelling of fixed oscillating water column wave power plant by a boundary element methods, Ocean Engineering,30, 2003

Gouaud, F.,Rey,V., Van Hooff,R. and Piazzola J., "Experimental study of the hydrodynamic performance of an onshore wave-power device in the presence of an underwater mound", Coastal Eng., 57, 996-1005 (2010).

Guazzelli, E.,Rey, V. and Belzons, M., "Higher-order Bragg reflection of gravity surface waves by periodic beds," J. Fluid Mech. 245, 301 (1992).

Holthuijsen, L. H., Herman A. and Booij, N.,"Phase-decoupled refraction-diffraction for spectral wave models," Coastal Eng. 49, 291-305, 2003.

Horikawa, *Nearshore Dynamics and Coastal Processes* (Tokyo press., 1988).

Jarry, N. , Etudes expérimentales et numériques de la propagation des vagues au-dessus de bathymétries complexes en milieu côtier, ,Ph-D,University of Toulon, France, december 2009 (in french).

Jarry, N., Rey, V. , Gouaud, F. and Lajoie, D., Gravity wave amplification and phase crest re-organization over a shoal, Nat. Hazards Earth Syst. Sci. 11, 789-796 (2011).

Longuet-Higgins, M. S. 1950 A theory of the origin of microseisms. Proc. R. Soc. Lond. Ser A 243, 1–35.

Magne, R., Belibassakis, K. A., Herbers, T. H. C., O Reilly, W. C., Ardhuin, F. and Rey, V. (2007), Evolution of surface gravity waves over a submarine canyon, J. Geophys. Res., 112, C01002, doi:101029/2005JC003035.

Mei, C. C., "The Applied Dynamics of Ocean Surface Waves", Ed. Wiley, New-York (1983).

Penney, W. G. and Price, A. T., "The diffraction theory of sea waves and the shelter afforded by breakwaters," Phil. Trans. Roy. Soc. London. Ser. A, **224**, pp 236-253 (1952).

Rey, V. "Propagation and local behaviour of normally incident gravity waves over varying topography," Eur. J. Mech. B: Fluids **11**, 213 (1992).

Rey, V., Belzons, M. and Guazzelli, E. 1992 Propagation of surface gravity waves over a rectangular submerged bar, J. Fluid Mech. 235, 453-479.

Rey, V. , Charland, J. and Touboul, J. 2014 Wave - current interaction in the presence of a three-dimensional bathymetry: Deep water wave focusing in opposing current conditions, Phys. Fluids, 26, 096601.

Rey, V., Arnaud, G., Touboul, J. and Belibassakis, K., "Water wave scattering by dense or sparse arrays of surface-piercing bodies by integral matching method", Applied Ocean Res., **75**, 132-142 (2018).

Rey, V. and Touboul, J., "Forces and moment on a horizontal plate due to regular and irregular waves in the presence of current", Applied Ocean Res., **33**, 88-99 (2011).

Touboul J. and Rey, V., 2012, Bottom pressure distribution due to wave scattering near a submerged obstacle. J. Fluid Mech.,702,444-459.