Waves and related processes in Geophysical Fluid Dynamics

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Lecture 2: Wave motions in the presence of boundaries, topography, mean flow, and at the equator

Introducing lateral boundaries and shelf
Lateral boundary : Kelvin waves
Shelf and related waves

Waves over topography/bathymetry far from lateral boundaries

Waves in outcropping flows

Equatorial waves
Equatorial waves in 1-layer model
Waves in 2-layer RSW with a rigid lid on the equatorial beta-plane

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Linearised RSW equations in a half-plane with a rectilinear coast at $x = 0$

Linearisation about the state of rest with $h = H_0$:

$$\begin{cases} u_t - fv + g\eta_x = 0, \\ v_t + fu + g\eta_y = 0, \\ \eta_t + H_0(u_x + v_y) = 0. \end{cases} \quad (1)$$

Free-slip boundary condition : $u|_{x=0} = 0 \rightarrow$ inhomogeneity in x.
Partial Fourier-transform

$$(u, v, \eta) = (\bar{u}_0(x), \bar{v}_0(x), \bar{\eta}_0(x))e^{i(ly - \omega t)} + c.c.$$

$\rightarrow$ wave equation :

$$\bar{\eta}_0'' + (\omega^2 - f^2 - gH_0 l^2)\bar{\eta}_0 = 0, \quad (2)$$

Wave solutions

- *free* inertia-gravity waves with

$$\omega^2 - f^2 - gH_0 l^2 \equiv k^2 > 0, \tag{3}$$

$$\bar{\eta}_0 \propto e^{\pm ikx}, \Rightarrow \omega^2 = f^2 + gH_0(k^2 + l^2). \tag{4}$$

- *trapped* at the boundary Kelvin waves with

$$\omega^2 - f^2 - l^2 \equiv -\kappa^2 < 0, \tag{5}$$

$$\bar{\eta}_0 \propto e^{-\kappa x}, \Rightarrow \kappa > 0. \tag{6}$$

Kelvin waves are dispersionless : $\kappa = -\frac{fl}{\omega} \Rightarrow$ $\omega^2 - f^2 - gH_0 l^2 + gH_0 \frac{f^2 l^2}{\omega^2} = 0$, and $\omega^2 = gH_0 l^2$. $\kappa > 0 \Rightarrow \omega = -\sqrt{gH_0}\, l$, and $\eta \propto e^{-\frac{x}{R_d}}$, $R_d = \frac{\sqrt{gH_0}}{f}$ - *Rossby deformation radius*.

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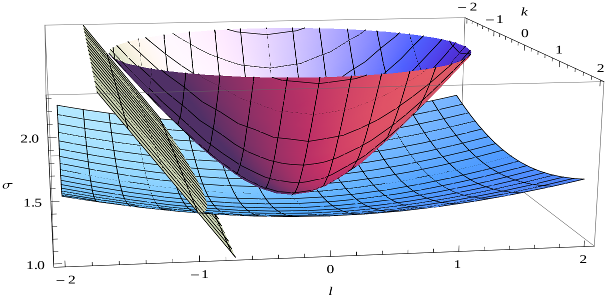
Dispersion relation for inertia-gravity and Kelvin waves in the 2-layer RSW model.



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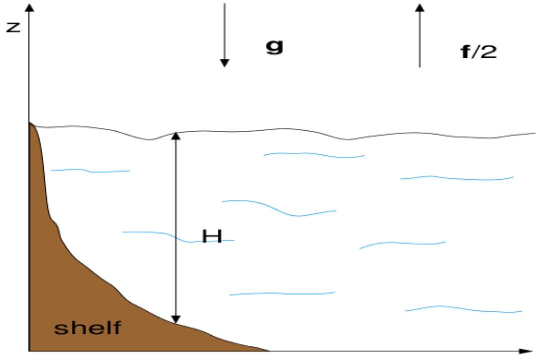
Lateral boundary with a shelf



RSW with topography :

$$\begin{cases} u_t + uu_x + vu_y - fv + g\eta_x = 0, \\ v_t + uv_x + vv_y + fu + g\eta_y = 0, \\ \eta_t + [[(H(x,y) + \eta)u]_x + [(H(x,y) + \eta)v]_y = 0. \end{cases} \quad (7)$$

One-dimensional topography $H = H(x)$, for simplicity.

Spectrum of small perturbations

Linearisation and elimination of variables $\rightarrow$

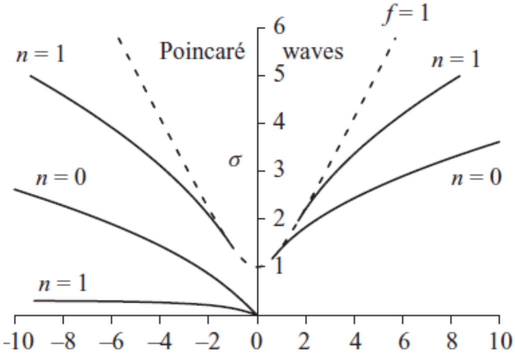
$$(gH\bar{\eta}_0')' + (\omega^2 - f^2 - gHl^2 - \frac{fl}{\omega}gH')\bar{\eta}_0 = 0. \qquad (8)$$

Remark : Ball's model $H(x) = H_0(1 - e^{-ax}) \Rightarrow$ hypergeometric equation.

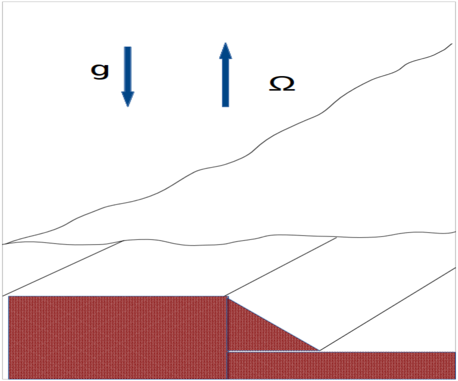
Spectrum for monotonous $H(x)$

- Single Kelvin wave with unidirectional propagation
- Discrete spectrum of sub-inertial unidirectional waves with $\omega < f$ (shelf waves). Both shelf and Kelvin waves propagate leftwards, looking at the coast,
- Discrete spectrum of supra-inertial waves with $\omega > f$ (edge waves), which may propagate in both directions along the coast,
- Continuous spectrum of incident/reflected inertia-gravity (Poincaré) waves.

Dispersion diagram in the Ball's model



Escarpment bathymetry



Same linearised equation for $\bar{\eta}_0$ with decay (trapped waves) or radiation (free waves) boundary conditions

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Example : linear escarpment

$$((H_m - x)\bar{\eta}_0')' + (\omega^2 - f^2 - l^2(H_m - x) + \frac{fl}{\omega})\bar{\eta}_0 = 0, \quad (9)$$

$H_m = \frac{H_+ + H_-}{2H_0}$ - non-dimensional mean depth. Can be explicitly solved in terms of confluent hypergeometric functions M and U. Geral solution of (9) is :

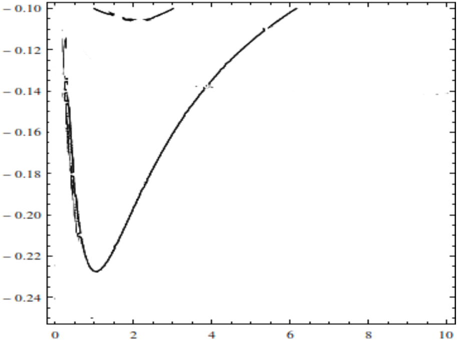
$$\begin{aligned}\bar{\eta}_0(x) &= C_1 U\left(-\frac{-fl - f^2\omega - l\omega + \omega^3}{2l\omega}, 1, 4l - 2lx\right) \\ &+ C_2 M\left(\frac{-fl - f^2\omega - l\omega + \omega^3}{2l\omega}, 1, 4l - 2lx\right) \quad (10)\end{aligned}$$

where $C_{1,2} =$ const. Should be matched to the solutions $\bar{\eta}_0(x) = C_\pm e^{\mp\sqrt{-p_\pm^2}}$ of the asymptotic equations at each side

$$gH_\pm \bar{\eta}_{0\pm}'' + (\omega^2 - f^2 - gH_\pm l^2)\bar{\eta}_{0\pm} = 0. \quad (11)$$

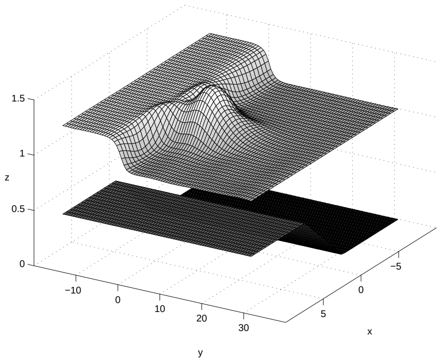
Conditions of solvability → dispersion relation.

Dispersion of trapped at the escarpment waves



Dispersion diagram for topographic waves trapped by a linear escarpment. Only two lowest modes with, respectively, zero and one nodes in the x - direction across the escarpment are shown. The waves can propagate only in the negative direction along the escarpment, i.e. leaving the shallower region on their right. Multiple curves with diminishing eigenfrequencies

Generation of topographic waves



Relaxation of a pressure front perpendicular to the bottom escarpment, as seen in the pressure (thickness) field. A packet of topographic waves starting along the escarpment is visible in a form of a bump.

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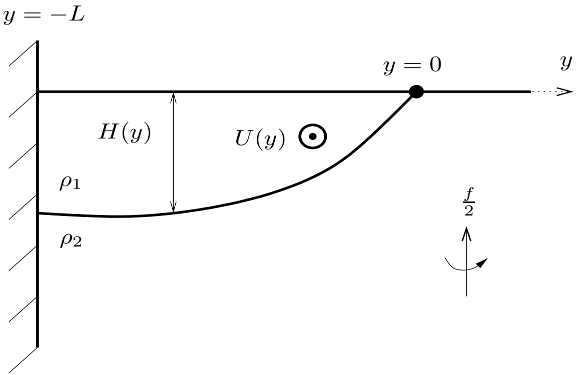
Outcropping coastal density current



Outcropping ⇒ non-trivial profile of the layer thickness H in a steady state ⇒ non-zero mean velocity via the geostrophic balance

$$U(y) = -\frac{g}{f} H_y(y) \quad (12)$$

LInearisation and boundary conditions

$$\begin{cases} u_t + Uu_x + vU_y - v = -\, h_x, \\ v_t + Uv_x + u = -\, h_y, \\ h_t + Uh_x = -(Hu_x + (Hv)_y). \end{cases} \quad (13)$$

Free-slip boundary condition at the coast : $v(-1) = 0$.
The outcropping line is a material line $\Rightarrow$:

$$H(y) + h(x, y, t)|_{y=Y_0} = 0, \quad \frac{dY_0}{dt} = v\Big|_{y=Y_0}. \quad (14)$$

$y = 0$ - location of the free streamline of the mean flow, $Y_0(x, t)$ - position of the perturbed free streamline, $\frac{d}{dt}$ - Lagrangian derivative. Linearised boundary conditions :

$$Y_0 = -\frac{h}{H_y}\Big|_{y=0}, \quad (15)$$

and continuity equation evaluated at $y = 0 \Rightarrow$ the only constraint to impose on the solutions of (13) is regularity at $y = 0$.

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Constant PV flows

PV of the mean flow in non-dimensional terms :

$$Q(y) = \frac{1 - U_y}{H(y)}, \quad U(y) = -H_y(y), \Rightarrow \tag{16}$$

$$H_{yy}(y) - Q(y)H(y) + 1 = 0, \quad H(0) = 0, \; H_y(0) = -U_0, \tag{17}$$

$U(0) = U_0$ is the mean flow velocity at the outcropping. Flows with constant : $Q(y) = Q_0 \neq 0$:

$$\begin{cases} H(y) = \frac{1}{Q_0}[1 - U_0\sqrt{Q_0}\, sinh(\sqrt{Q_0} y) - \; cosh(\sqrt{Q_0} y)], \\ U(y) = U_0\, cosh(\sqrt{Q_0} y) + \frac{1}{\sqrt{Q_0}}\, sinh(\sqrt{Q_0} y). \end{cases} \tag{18}$$

Advantage : for $(u, v, h) = (\bar{u}(y), \bar{v}(y), \bar{h}(y))e^{ik(x-ct)} + c.c.$, the wave equation does not have singularity, which is otherwise the case, at critical levels y_c : $U(y_c) - c = 0$.

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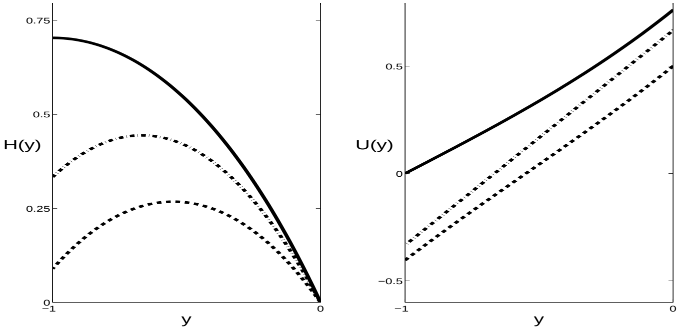
Examples of constant PV flows



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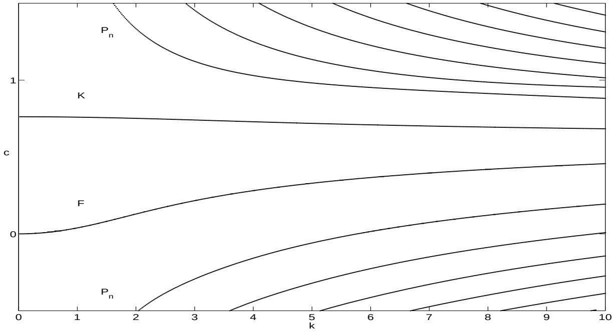
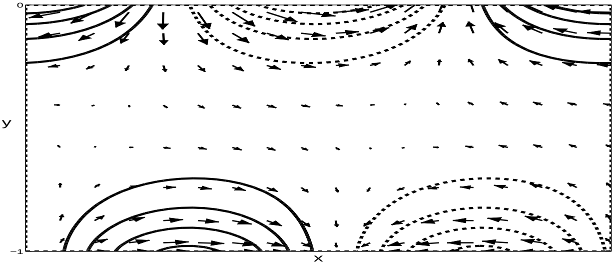
Dispersion diagram



Dispersion diagram for waves in the flow with $Q_0 = 1$. K - coastal Kelvin wave, F - frontal wave, P_n - Poincaré (inertia-gravity) wave, n - number of nodes of the mode in the span-wise direction.

Phase portraits of Kelvin and Frontal waves



Pressure (contours) and velocity (arrows) anomalies of Kelvin (bottom) and frontal (top) waves propagating over a uniform PV flow flow with $Q_0 = 1$.

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Specifics of equatorial tangent plane

Tangent plane at the equator $\Rightarrow$ rotation of the planet is parallel to the plane. 1-layer RSW model on the equatorial beta-plane - no f_0 :

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \beta y \, \hat{\mathbf{z}} \wedge \mathbf{v} + g \nabla h = 0 \,, \\ \partial_t h + \nabla \cdot (\mathbf{v} h) = 0 \,. \end{cases} \tag{19}$$

Decay boundary conditions in y (confinement in the equatorial region). Linearised non-dimensional equations :

$$\begin{cases} u_t - y\, v + h_x &= 0, \\ v_t + y\, u + h_y &= 0, \\ h_t + u_x + v_y &= 0. \end{cases} \tag{20}$$

Explicit dependence on y !

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Gauss - Hermite basis

Change of dependent variables

$$f = \frac{1}{2}(u + h); \quad g = \frac{1}{2}(u - h). \tag{21}$$

$$\begin{cases} f_t + f_x + \frac{1}{2}(v_y - yv) = & 0, \\ g_t - g_x - \frac{1}{2}(v_y + yv) = & 0, \\ v_t + y(f + g) + (f - g)_y = & 0, \end{cases} \tag{22}$$

appearance of operators $\partial_y \pm y$. $\exists$ a set of orthonormal functions such that :

$$\phi_n' + y\phi_n = \sqrt{2n}\phi_{n-1}, \quad \phi_n' - y\phi_n = -\sqrt{(2n+1)}\phi_{n+1}. \tag{23}$$

Gauss-Hermite functions, H_n - Hermite polynomials

$$\phi_n(y) = \frac{H_n(y)e^{-\frac{y^2}{2}}}{\sqrt{2^n n! \sqrt{\pi}}}, \tag{24}$$

$$\phi_n''(y) + (2n + 1 - y^2)\phi_n(y) = 0, \tag{25}$$

with decay boundary conditions.

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Special solutions : Kelvin wave

Particular solution with $v \equiv 0$:

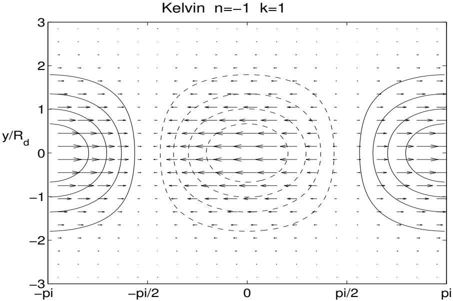
$$f_t + f_x = 0,\ g_t - g_x = 0,\ \Rightarrow f = F(x-t, y),\ g = G(x+t, y),$$

$$y(f+g) + (f-g)_y = 0,\ \Rightarrow F \propto e^{-\frac{y^2}{2}},\ G \propto e^{+\frac{y^2}{2}}.$$

Decay boundary conditions impose $G \equiv 0 \Rightarrow$

$$u = F_0(x-t)e^{-\frac{y^2}{2}};\ \ h = F_0(x-t)e^{-\frac{y^2}{2}};\ \ v = 0. \quad (26)$$

Equatorial Kelvin wave with unique sense of propagation, eastwards, and no dispersion.



Special solutions : Yanai waves

Another particular solution with $g = 0$, $f \neq 0$, $v \neq 0 \Rightarrow$

$$\begin{cases} f_t + f_x + \frac{1}{2}(v_y - yv) = 0, \\ v_y + yv = 0, \\ v_t + yf + f_y = 0, \end{cases} \tag{27}$$

Separation of variables :

$$v = v_0(x,t)\,\phi_0(y), \quad f = F_1(x,t)\,\phi_1(y) \;\Rightarrow \tag{28}$$

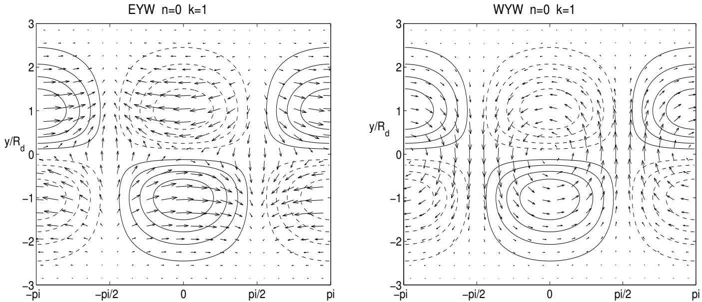
equations with constant coefficients for $F_1(x,t),\ v_0(x,t)$:

$$F_{1_t} + F_{1_x} - \frac{1}{\sqrt{2}} v_0 = 0, \quad v_{0_t} + \sqrt{2} F_1 = 0. \tag{29}$$

Looking for wave solutions $\propto e^{i(\omega t - kx)}$ we get the dispersion relation :

$$\omega = \frac{k}{2} \pm \sqrt{\frac{k^2}{4} + 1}. \tag{30}$$

Phase portraits of Yanai waves



Pressure (contours) and velocity (arrows) distribution in the equatorial eastward- (left panel) and westward- (right panel) propagating Yanai waves with zonal wavenumber $k = 1$.

General solution : inertia-gravity and Rossby waves

Elimination of u and h (or f and g) in favour of v :

$$\partial_t \left(\nabla^2 v - y^2 v - \partial_{tt} v \right) + \partial_x v = 0. \quad (31)$$

Expansion of v in ϕ_n : $v = \sum_n v_n(x,t)\phi_n(y)$ gives :

$$\partial_t \left[\partial^2_{xx} v_n - (2n+1) v_n - \partial^2_{tt} v_n \right] + \partial_x v_n = 0. \quad (32)$$

After Fourier-transformation $\tilde{v}_n(k,t) = \int dx e^{-ikx} v_n(x,t) + c.c.$ we get

$$\partial^3_{ttt} \tilde{v}_n + (k^2 + 2n + 1)\partial_t \tilde{v}_n - ik\tilde{v}_n = 0. \quad (33)$$

General solution

$$\tilde{v}_n = v_{n_1}(k) e^{-i\omega_{n_1} t} + v_{n_2}(k) e^{-i\omega_{n_2} t} + v_{n_3}(k) e^{-i\omega_{n_3} t}, \quad (34)$$

where ω_{n_α}, $\alpha = 1, 2, 3$ are roots of the dispersion relation :

$$\omega^3_{n_\alpha} - (k^2 + 2n + 1)\omega_{n_\alpha} - k = 0. \quad (35)$$

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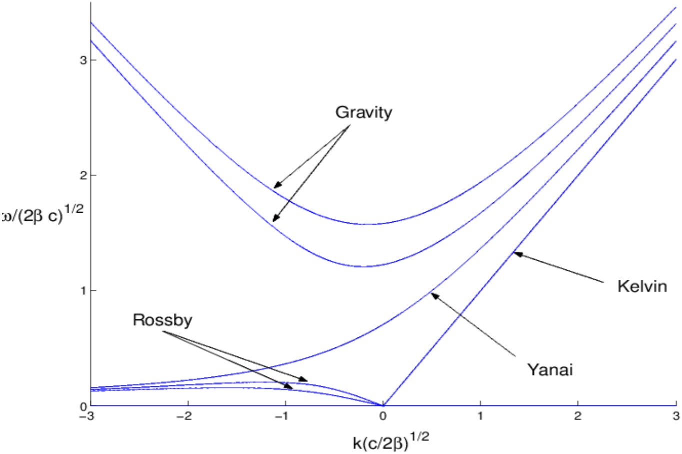

Dispersion diagram for equatorial waves in the 1-layer RSW. Only two lowest meridional modes for Rossby and inertia-gravity waves are shown.

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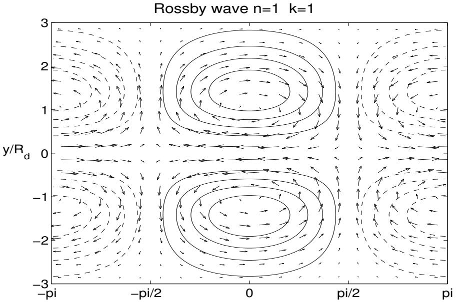
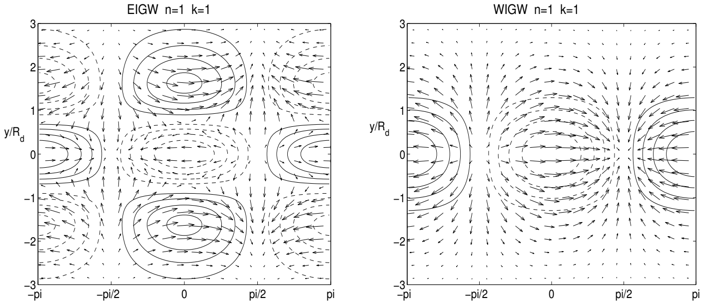
Phase portrait of a Rossby wave



Pressure (contours) and velocity (arrows) distribution in the equatorial Rossby wave with zonal wavenumber $k = 1$.

Phase portraits of inertia-gravity waves



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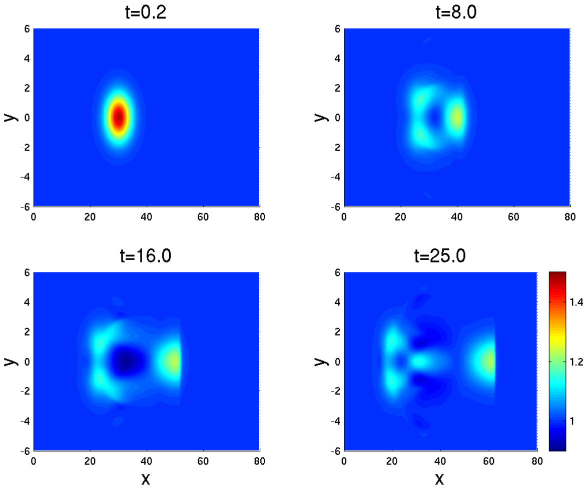
Generation of Kelvin and Rossy waves by pressure anomaly : numerical simuations



Relaxation of a pressure anomaly of large zonal scale at the equator, with formation of Rossby and Kelvin waves .

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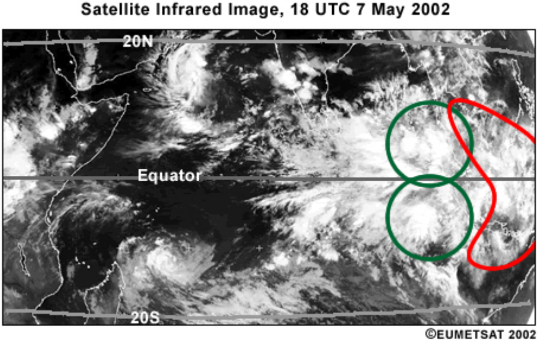
Kelvin and Rossy in satellite observatons



Twin cyclones and Kelvin front at the equator.

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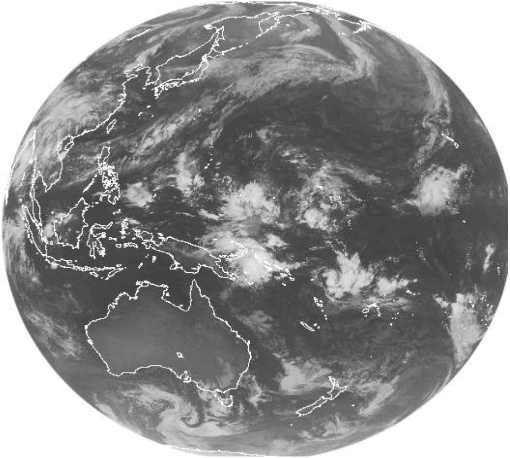
Equatorial Rossby wave in real life



Symmetric with respect to equator twin depression visible in the cloud cover in a satellite image and associated with an equatorial Rossby wave.

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Equations of motion and barotropic-baroclinic decomposition

Standard 2-layer model ones with a replacement $f \to \beta y$:

$$\begin{cases} \partial_t \mathbf{v}_i + \mathbf{v}_i \cdot \nabla \mathbf{v}_i + \beta y \hat{\mathbf{z}} \wedge \mathbf{v}_i + \frac{1}{\rho_i} \nabla \pi_i = 0 \,, \; i = 1, 2; \\ \partial_t h_i + \nabla \cdot (h_i \mathbf{v}_i) = 0, \; i = 1, 2; \\ \pi_1 = \pi_2 + \rho_1 g' h_1 \,, \; g' = g \frac{\rho_1 - \rho_2}{\rho_1}, \;\; h_1 + h_2 = H. \end{cases} \tag{36}$$

Barotropic and baroclinic components of velocity :

$$\mathbf{v}_{bt} = \frac{h_1 \mathbf{v}_1 + h_2 \mathbf{v}_2}{H}, \;\; \mathbf{v}_{bc} = \mathbf{v}_1 - \mathbf{v}_2. \tag{37}$$

Rigid-lid constraint $h_1 + h_2 =$ const and continuity equations $\Rightarrow$ incompressibility constraint :

$$\nabla \cdot (h_1 \mathbf{v}_1 + h_2 \mathbf{v}_2) = H \nabla \cdot \mathbf{v}_{bt} = 0, \Rightarrow \tag{38}$$

barotropic stream-function ψ :

$$\mathbf{v}_{bt} = \hat{\mathbf{z}} \wedge \nabla \psi. \tag{39}$$

Lecture 2: Wave motions in the presence of boundaries, topography, mean flow, and at the equator
Introducing lateral boundaries and shelf
Lateral boundary : Kelvin waves
Shelf and related waves
Waves over topography/bathymetry far from lateral boundaries
Waves in outcropping flows
Equatorial waves
Equatorial waves in 1-layer model
Waves in 2-layer RSW with a rigid lid on the equatorial beta-plane
Résumé

Equations in barotropic/baroclinic components

$$\nabla^2\psi_t + \psi_x = \epsilon\left[-J(\psi, \nabla^2\psi) - s(\partial_{xx} - \partial_{yy})\left[(1+\epsilon qh)(uv)\right] + s\partial_{xy}\left(u^2 - v^2\right)\right], \quad (40)$$

$$\mathbf{v}_t + \nabla h + y\hat{\mathbf{z}} \times \mathbf{v} = \epsilon\left[-J(\psi, \mathbf{v}) + \mathbf{v}\cdot\nabla(\hat{\mathbf{z}} \times \nabla\psi) - q\mathbf{v}\cdot\nabla\mathbf{v} + \epsilon s\left(2h\mathbf{v}\cdot\nabla\mathbf{v} + \mathbf{v}\,\mathbf{v}\cdot\nabla h\right)\right], \quad (41)$$

$$h_t + \nabla\cdot\mathbf{v} = \epsilon\left[-J(\psi, h) + \epsilon s\nabla\cdot\left(h^2\mathbf{v}\right) - q\nabla\cdot(\mathbf{v}h)\right], \quad (42)$$

ϵ is the Rossby number, and $q = (H - 2H_1)/H$ and $s = H_1 H_2 / H^2$.

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Linearised equations and wave spectrum

Linearisation :

$$\begin{cases} \nabla^2\psi_t + \psi_x = 0, \\ \mathbf{v}_t + \nabla h + y\hat{\mathbf{z}} \times \mathbf{v} = 0, \\ h_t + \nabla \cdot \mathbf{v} = 0. \end{cases} \quad (43)$$

Spectrum

- Trapped baroclinic waves :

$$(u, v, h) = (iU_n(y), V_n(y), iH_n(y))\, Ae^{i(kx-\omega_n)t} + c.c., \quad (44)$$

$$\omega_n^3 - (k^2 + 2n + 1)\omega_n - k = 0; \ n = -1, 0, 1, 2, \dots, \quad (45)$$

- Barotropic "free" Rossby waves,

$$\psi_0 = A_\psi e^{i(kx-\omega t+ly)} + c.c., \quad (46)$$

$$\omega = -k/(k^2 + l^2), \quad (47)$$

What have we seen :

- Topography, coasts and outcroppings lead to appearance of new class of trapped waves
- Same for the equator
- Two main kinds of wave-guide modes, both unidirectional :
  1. non-dispersive no-PV Kelvin waves,
  2. dispersive, PV-bearing Rossby waves.
- Kelvin waves fill the spectral gap

What we have not seen :

Wave couplings/interactions - coming up !

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