Elliptic problems in smooth and non-smooth domains

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Abstract

We are interested here in questions related to the reg- ularity of solutions of elliptic problems with Dirichlet or Neumann boundary condition (see ([1]). For the last 20 years, lots of work has been concerned with questions when Ω is a Lipschitz domain. We give here some complements for the case of the Laplacian (see [3]), the Bilaplacian ([2],[6]) and the operator $\operatorname{div}(A\nabla)$ (see ([5]), when A is a matrix or a function, and we extend this study to obtain other regularity results for domains having an adequate regularity. Using the duality method, we will then revisit the work of Lions–Magenes [4], concerning the so-called very weak solutions, when the data are less regular. Thanks to the interpolation theory, it permits us to extend the classes of solutions and then to obtain new results of regularity.

Keywords: elliptic problems, Laplacian, Bilaplacian, very weak solutions, interpolation theory

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