Solutions of the heat equation with large initial data

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Abstract

I will discuss the behaviour of solutions of the heat equation for initial data that is allowed to be large 'at infinity', namely u_0 such that $\int_{\mathbb{R}^d} \mathrm{e}^{-\varepsilon|x|^2} u_0(x)\,\mathrm{d}x < \infty$ for some $\varepsilon > 0$ ('$u_0 \in L^1_\varepsilon$'). Such classes are natural when considering solutions of the heat equation that can be given in terms of the heat kernel. Solutions arising from such initial data can blow up in finite time; and if $u_0 \in L^1_\varepsilon$ for every $\varepsilon > 0$ then, while the solution exists for all time, the behaviour of the solution can be 'wild' as $t \to \infty$. The results are readily transferred to a class of solutions of the Stokes equations $u_t - \Delta u + \nabla p = 0$, $\nabla \cdot u = 0$.

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Keywords: Heat equation; Stokes equations; blow up.