The Interface Control Domain Decomposition (ICDD) in Multidimensional Hydrodynamics

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Abstract

The 3D Navier-Stokes system of equations is the most suitable mathematical model for describing the wide range of physical phenomena of different nature, space dimension and time scales which are involved in the simulation of free surface flows. Because of the complexity of Navier-Stokes equations, some models in lower space dimensions have been derived from them. The aim consists in using simpler models where the space dimensions and the geometry of the problem make it possible: examples are represented by the 2D shallow water system, or, in the 1D case, by the Saint-Venant system. However, the simulation of the motion of water in a complex hydrodynamic configuration needs to take into account the complexity of the problem: hence, the idea is to reduce the computational cost by solving the most expensive model only in some parts of the computational domain, i.e. where it is strictly necessary. In this paper we consider, in particular, the problem of the coupling between 1D and 2D shallow

water systems. To this aim we introduce the so-called interface control domain decomposition method, that represents a valid alternative to more classical methods for the resolution of heterogeneous problems. In our case we generalize the situation of heterogeneous differential problems by introducing an heterogeneity in space. The basic idea of this method consists in splitting the original domain into two (or more) overlapping sub-domains and in introducing some variables on the interfaces, that minimize a suitable cost functional and play the role of boundary data for the local differential subproblems at the interfaces.
In this paper we introduce the theoretical aspects of the resolution of the 1D-2D SWE coupling by ICDD and report the obtained numerical results.

Keywords: shallow water, multidimensional coupling, domain decomposition, optimal control, interface control, elliptic problems, finite element method

References

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